Tutorial on the API Standard Paragraphs Covering Rotor Dynamics and Balancing: An Introduction to Lateral Critical and Train Torsional Analysis and Rotor Balancing

API PUBLICATION 684
FIRST EDITION, FEBRUARY 1996

American
Petroleum
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Manufacturing, Distribution and Marketing Department

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SECTION 1—ROTOR DYNAMICS: LATERAL CRITICAL ANALYSIS

1.1 Scope

This document is intended to describe, discuss, and clarify Section 2.8 of the API *Standard Paragraphs* (SP) (Revision 20). Section 2.8 outlines the complete rotor dynamics acceptance program designed by API to insure equipment mechanical reliability. Before discussion of the standard paragraphs proceeds, however, background material on the fundamentals of rotor dynamics (including terminology) and rotor modeling is presented for those unfamiliar with the subject. This information is not intended to be the primary source of information for this complex subject but, rather, is offered as an introduction to the major aspects of rotating equipment vibrations that are addressed during a typical lateral dynamics analysis.

1.2 Introduction to Rotor Dynamics

1.2.1 GENERAL

The ultimate mechanical reliability of rotating equipment depends heavily upon decisions made by both the purchaser and vendor prior to equipment manufacture. Units that are designed using sophisticated computer-aided engineering methods will be less problematic than units designed without the benefit of such analysis. Even if the purchaser of rotating equipment contracts the vendor to perform mechanical acceptance tests prior to delivery and installation the discovery of design-related problems during these tests will likely compromise the planned cost of the unit and/or its delivery schedule. For this reason, specifying a mechanical acceptance test without also requiring a design analysis and review prior to construction may force a purchaser to accept equipment that will prove problematic after installation.

In order to aid turbomachine purchasers, the American Petroleum Institute's Subcommittee on Mechanical Equipment has produced a series of specifications that define mechanical acceptance criteria for new rotating equipment. Experience accumulated by turbomachine purchasers over the past ten years indicates that if the API standards are properly applied, the user can be reasonably assured that the installed unit is fundamentally reliable and will, barring problems with the installation and operator misuse, provide acceptable service over its design life. The backbone of these individual equipment specifications is formed by the API *Standard Paragraphs*, those specifications that are generally applicable to all types of rotating equipment. The rotating equipment specifications published by API (for example, API Standard 617—*Centrifugal Compressors for General Refinery Service*) are composed of applicable standard paragraphs and information that are applicable only to the type of unit considered in the standard.

The science of rotor dynamics has been extensively developed as a consequence of the realization by industry that the lateral dynamics characteristics and behavior of rotating equipment profoundly impacts plant operation. For example, a unit that possesses a highly amplified first critical speed may experience an increase in the clearance of inter-stage labyrinth seals during start-up. Unit aero-thermodynamic efficiency naturally suffers when the labyrinth seal clearances increase. The latest revision of the API *Standard Paragraphs* acknowledges the importance of rotor dynamics design analysis by incorporating acceptance criteria for a proposed or premanufactured design, in addition to test stand acceptance criteria for the completed units. Older API *Standard Paragraphs* emphasized only the measured test lateral vibrations of the completed equipment. The older experimental approach did not take advantage of the sophisticated computer analysis tools developed over the past twenty years for calculating the rotor dynamic characteristics of a given design. Most rotating equipment vendors have the capability to accurately calculate the rotor dynamic characteristics of a design prior to manufacture using standard rotor system modeling methods and associated computer software tools. Such characteristics include the resonance frequencies of the spinning shaft, the damped response of the rotating element to a specified unbalance over the expected speed range of operation, and the sensitivity of the unit to destabilizing forces generated by bearings, seals, and impellers. Experience with measured vibration behavior of units possessing similar designs aids the analyst in modeling a proposed machine because the influence of aerodynamics and seals are often difficult to accurately quantify solely by theoretical means prior to construction and operation of a unit. Construction and testing of many similar units enable an analyst to better understand the limitations of a particular analysis and to compensate for the limitations by using empirical data. The process of *model tuning* using empirically gathered data will generally permit extremely accurate predictions of calculated rotor dynamic characteristics if the proposed design falls inside or is near the envelope of prior experience.

Section 2.8 of the API *Standard Paragraphs* addresses the lateral rotor dynamics of turbomachinery. These standard paragraphs outline a design and machine evaluation program that if properly implemented by the equipment manufacturer and the end-user will ensure that the vibrations generated by the unit after installation are within acceptable limits during normal operation. The three-phase program outlined in the standard paragraphs consists of the following:

a. Modeling and analysis of the proposed or premanufactured design: The analysis to be conducted for the proposed design is described, including significant features that must be incorporated into the computer model to ensure that the model accurately portrays the dynamic behavior of the assembled unit.
b. Evaluation of the proposed design: acceptance/rejection criteria are offered for the proposed design.
c. Shop testing and evaluation of the assembled machine: Acceptance/rejection criteria are offered for the completed machine, based on rotor vibrations measured during the mechanical acceptance test.

Proper implementation of the program outlined in the applicable API standard ensures that turbomachine vendors will eliminate most problems during the unit's design, prior to manufacture. As the revised API acceptance criteria for rotor-bearing system designs has been embraced by turbomachine vendors, the discovery of design-related vibration problems during the unit test has decreased substantially. The responsibility of the purchaser is to ensure the proper implementation of the plan by the equipment manufacturer as outlined by the specific API standards.

The introductory material is divided into the following three parts:

a. A partial listing of basic terminology with brief definitions and relevant discussion.
b. A tutorial on the fundamental concepts of rotating equipment vibrations.
c. A discussion of the typical steps in a generic rotor-dynamics design analysis.

1.2.2 DEFINITION OF TERMS

1.2.2.1 *Amplification factor (AF)* is a measure of a rotor-bearing system's vibration sensitivity to unbalance when operated in the vicinity of one of its lateral critical speeds. A high amplification factor ($AF >> 10$) indicates that rotor vibration during operation near a critical speed could be considerable and that critical clearance components such as labyrinth seals and bearings may rub stationary elements during such periods of high vibration. If the rotor is designed to operate above a highly amplified lateral critical speed, then the unit's design might be considered unacceptable. A low amplification factor ($AF < 5$) indicates that the system is not sensitive to unbalance when operating in the vicinity of the associated critical speed. The effect of the amplification factor on rotor response near the associated critical speed is presented in Figure 1-1. The method of calculating amplification factor from damped response calculations or vibration measurements is also presented in this figure.

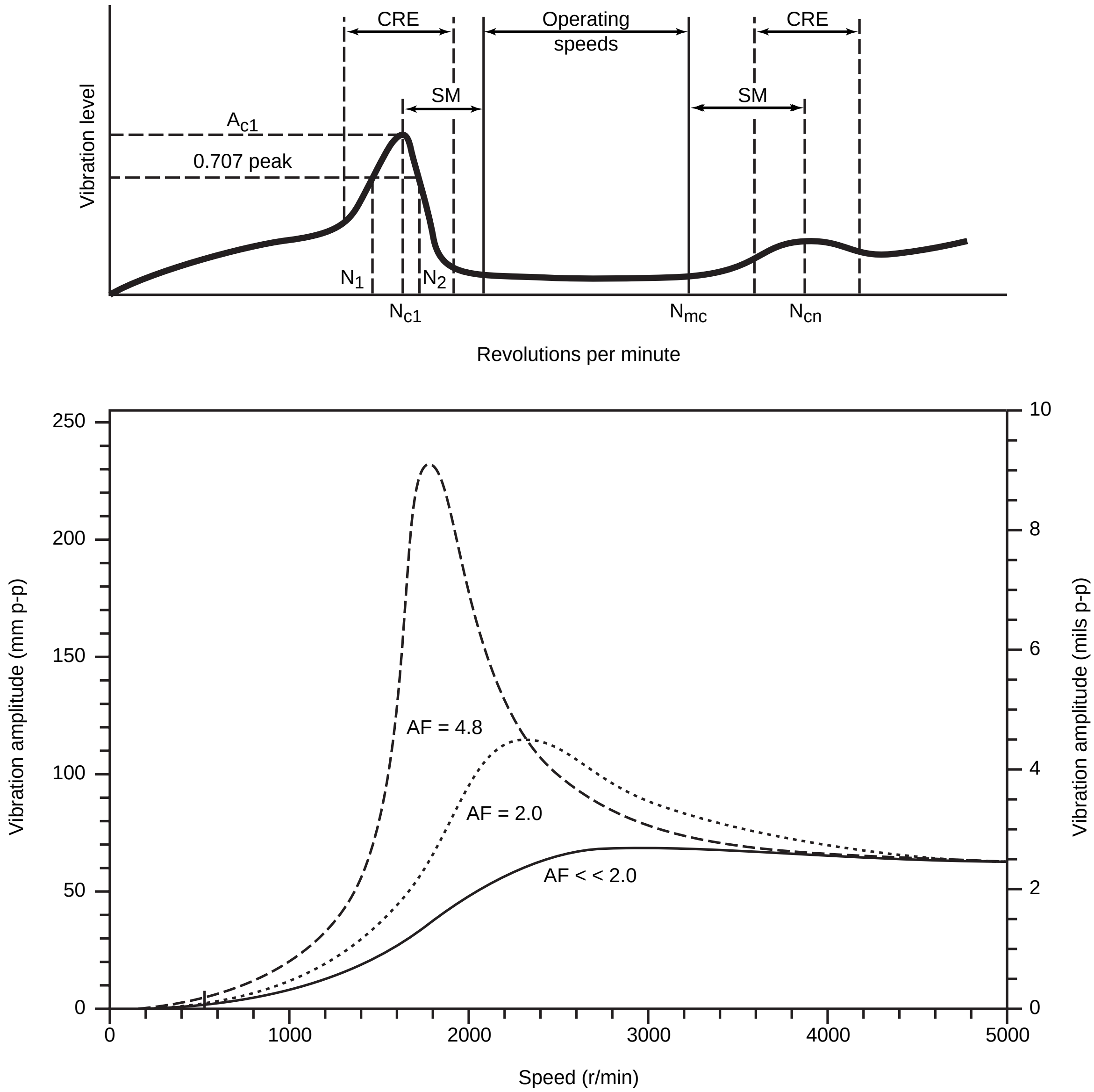
1.2.2.2 *A Bodé plot* is a graphical display of a rotor's synchronous vibration amplitude and phase angle as a function of shaft rotation speed. A Bodé plot is the typical result of a rotor damped unbalance response analysis. According to the most recent revisions of the API *Standard Paragraphs,* a rotor system's critical speeds are determined using response amplitude information presented in the Bodé plot. A sample Bodé plot is presented in Figure 1-2.

1.2.2.3 *A Campbell diagram* is a graphical presentation of the natural frequencies of either the equipment train or an individual unit and potential harmonic excitation frequencies. This graph clearly indicates acceptable train operating speeds where the train or individual unit may operate so that potential vibration excitation mechanisms (including, but not limited to, unbalance) do not interfere with important resonance frequencies: lateral critical speeds, blade resonance frequencies, or torsional natural frequencies. See Figure 1-3 and Figure 1-4 for examples.

1.2.2.4 *Critical speed* is defined in the standard paragraphs as a shaft rotational speed that corresponds to a non-critically damped ($AF > 2.5$) rotor system resonance frequency. According to API Standard Paragraph 2.8.1.3 (see Figure 1), the frequency location of the critical speed is defined as the frequency of the peak vibration response as defined by the Bodé plot, resulting from a damped unbalance response analysis and shop test data.

1.2.2.5 *Critical speed of concern* is any critical speed to which the acceptance criteria of this standard is applicable. In general, critical speeds of concern are (a) any critical speed below the operating speed range of the unit, (b) any critical speed in the operating speed range of a unit, and (c) the first critical speed above unit MCOS (maximum continuous operating speed). In special cases, other critical speeds may also be critical speeds of concern: for example, critical speeds that are integer multiples of electric line frequency or other excitation mechanisms. See API *Standard Paragraphs*, 2.8.1.5 for other potential critical speed excitation mechanisms.

1.2.2.6 *An undamped critical speed map* is a graph of a rotor's undamped critical speeds calculated as a function of the combined bearing/support stiffness. Typically, only the first three undamped critical speeds of the rotating element are presented in this plot. Calculated bearing stiffness is often cross-plotted on the critical speed map to help identify rotor dynamic characteristics of the actual rotor-bearing system. The critical speed map is not used to calculate the rotor's critical speeds because effects such as bearing and seal damping, aerodynamic cross-coupling, and the like are not



N_{c1} = Rotor first critical, center frequency, cycles per minute.
N_{cn} = Critical speed, nth.
N_{mc} = Maximum continuous speed, 105 percent.
N_1 = Initial (lesser) speed at 0.707 × peak amplitude (critical).
N_2 = Final (greater) speed at 0.707 × peak amplitude (critical).
$N_2 - N_1$ = Peak width at the half-power point.
AF = Amplification factor:
$= \dfrac{N_{c1}}{N_2 - N_1}$
SM = Separation margin.
CRE = Critical response envelope.
A_{c1} = Amplitude at N_{c1}.
A_{cn} = Amplitude at N_{cn}.

Figure 1-1—Evaluating Amplification Factors (AFs) from Speed-Amplitude Plots

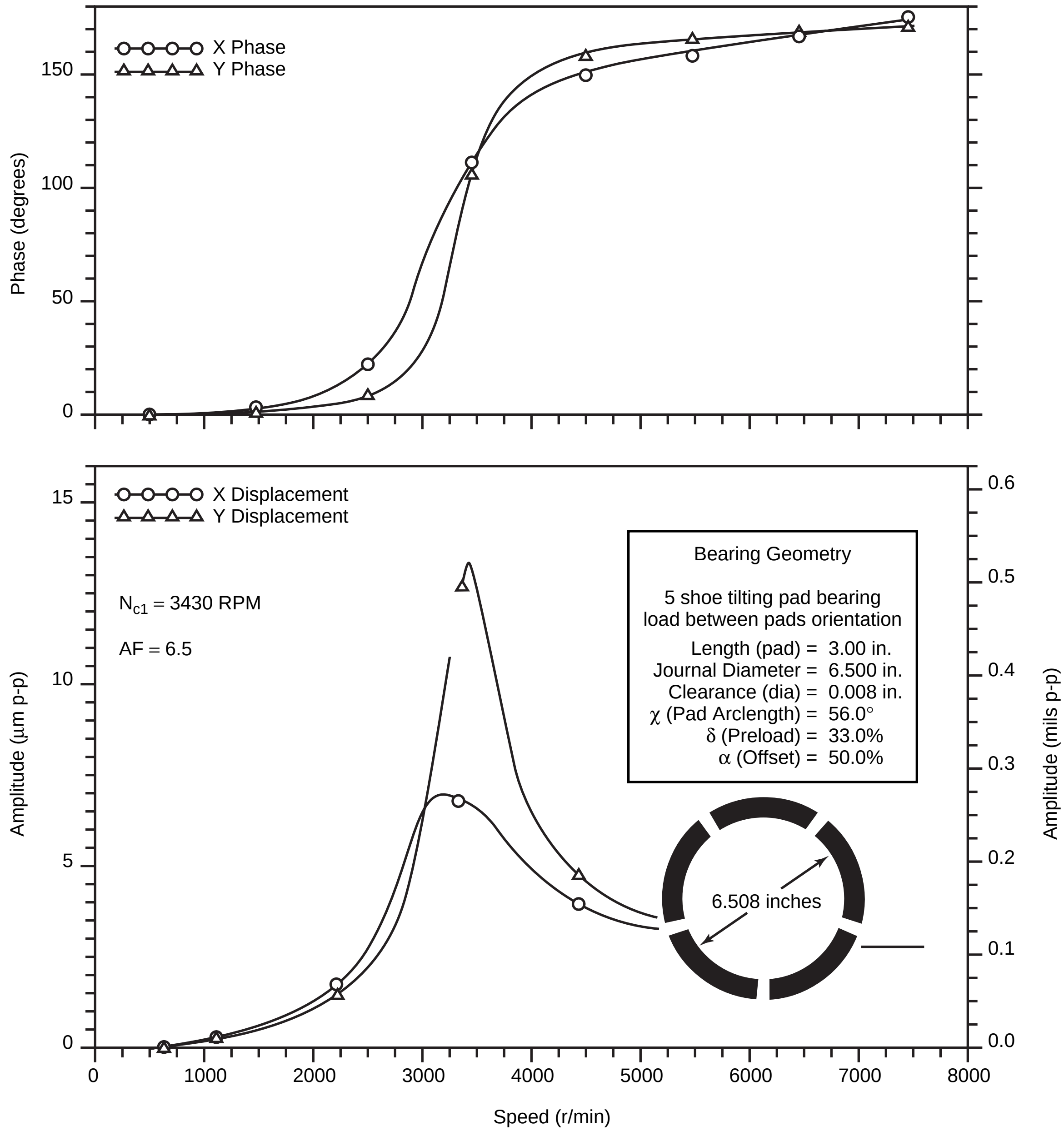


Figure 1-2—A Sample Bode Plot: Calculated Damped Unbalance and Phase Responses of an Eight-Stage 12 MW(16,000 HP) Steam Turbine

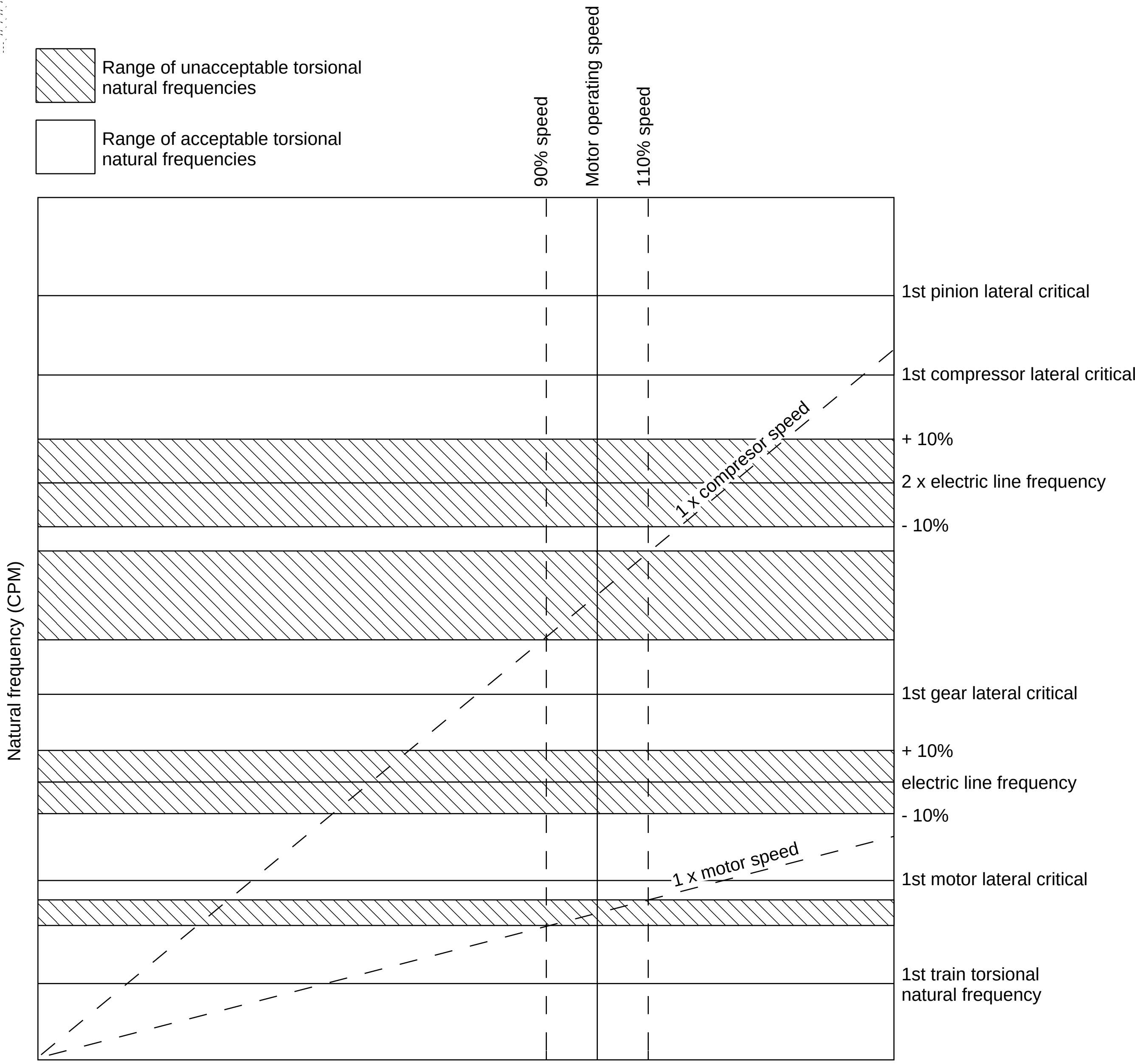


Figure 1-3—Sample Train Campbell Diagram for a Typical Motor-Gear-Compressor Train

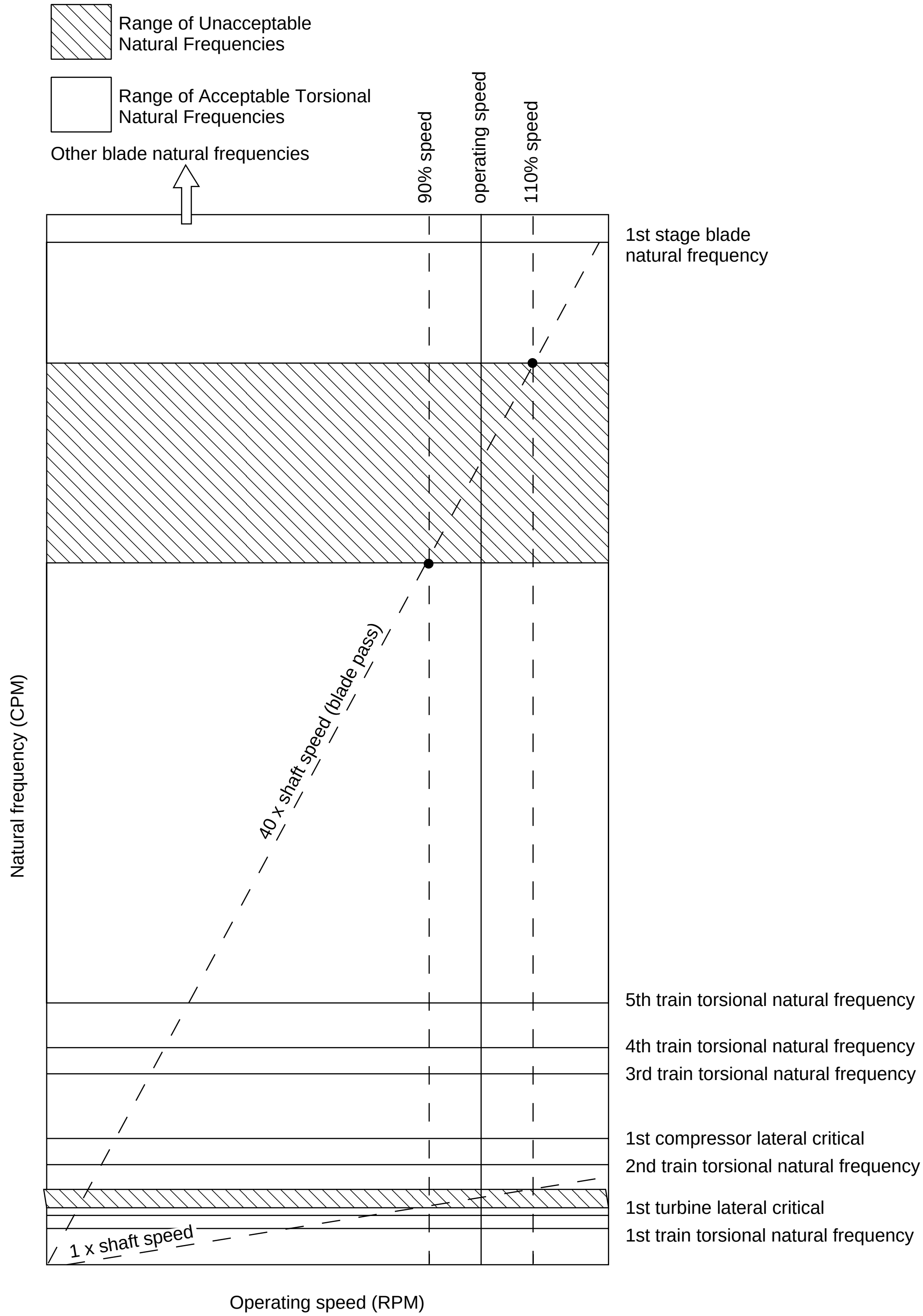


Figure 1-4—Sample Train Campbell Diagram for a Typical Turbine-Compressor Train

accounted for in an undamped critical speed analysis. Figure 1-5 displays a sample critical speed map.

1.2.2.7 *A damped unbalanced response analysis* is a calculation of the rotor's response to a set of applied unbalances. The applied unbalance excites the rotor synchronously, so the rotor's response to the applied unbalance will occur at the frequency of the shaft's rotation speed. The damped unbalance response analysis should account for all applied steady state linearized forces (bearing and seal stiffness and viscous damping, support effects, and others) and is used to predict the critical speed characteristics of a machine. Results of this analysis are typically presented in Bodé plots.

1.2.2.8 *Damping* is a property of a dynamic system by which mechanical energy is removed. Damping is important in controlling rotor vibration characteristics and is usually provided by viscous dissipation in fluid film bearings, floating ring oil seals, and so forth. Other sources of energy dissipation present in rotating systems, such as material damping, friction, and so forth, are generally considered negligible relative to the damping provided by the bearings and oil film seals.

1.2.2.9 *Frequency* is the number of cycles of a repetitive motion within a unit of time. Frequency is typically calculated as the reciprocal of the period of the repeating motion. Frequency is generally expressed as cycles per second (cps/Hertz) or cycles per minute (cpm). The latter units, cpm, afford ready comparison of the measured vibration with shaft rotating frequency.

1.2.2.10 *The logarithmic decrement (log dec)* is the natural logarithm of the ratio of any two successive amplitude peaks in a free harmonic vibration; the log dec is mathematically derived from the real part of the damped system eigenvalues calculated during a rotor dynamic stability analysis. The log dec provides a measure of rotor system stability. A

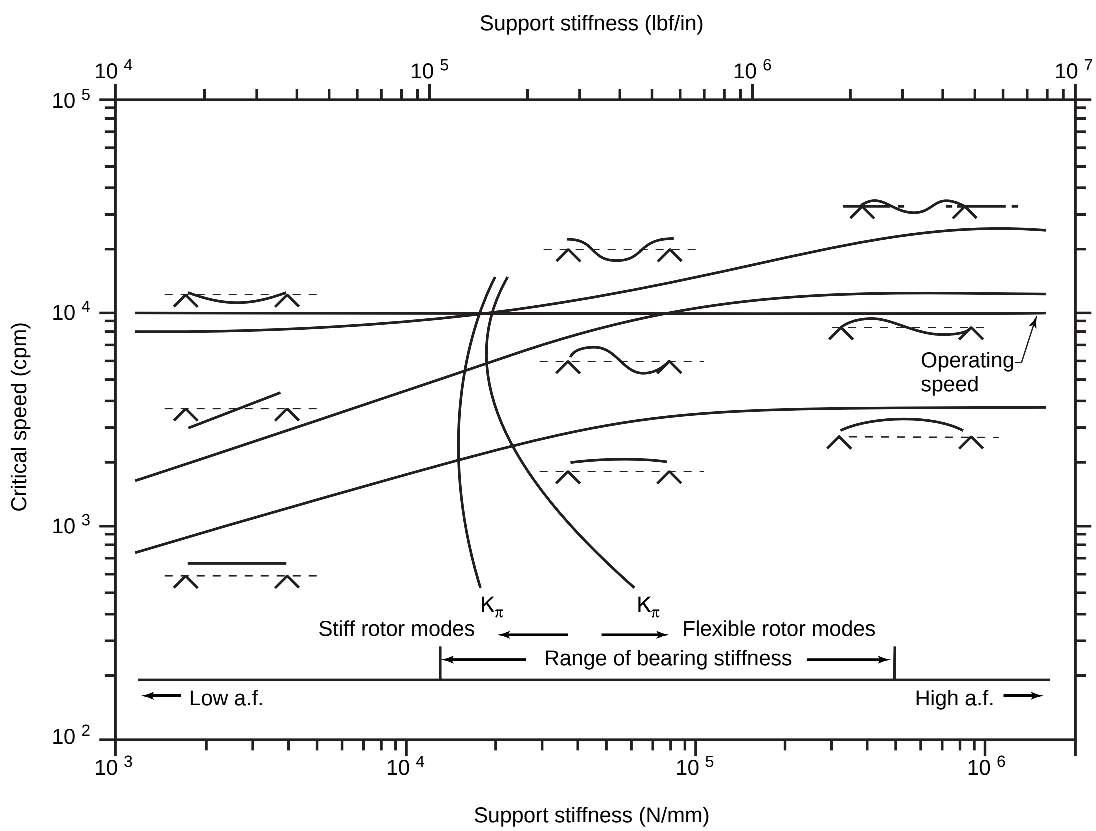


Figure 1-5—Undamped Critical Speed Map

positive log dec indicates that the system is stable while a negative log dec indicates the system is unstable.

1.2.2.11 *A mode shape* is the deflected shape of a rotor calculated at the critical speed during a damped unbalance response analysis. Each critical speed of a rotor will have a different mode shape. Mode shapes for the lowest frequency critical speeds are generally similar: the fundamental mode is usually a translational (*bounce*) mode, the second mode is usually a conical (*rocking*) mode, while the third mode is a *U-shaped mode*, often called the *first bending mode*. The general appearance of the mode shape corresponding to a critical speed can be dramatically altered when the stiffness of the bearings is substantially changed relative to the shaft bending stiffness. For example, Figure 1-6 displays the first three mode shapes of a typical rotor whose bearings are much softer (less stiff) than the bending stiffness of the shaft and for the rotor with bearings that are much stiffer than the shaft.

1.2.2.12 *Natural frequency* is synonymous with *resonant frequency* (see 1.2.2.14)

1.2.2.13 *The phase angle* is the angular difference between a shaft reference mark and the maximum shaft displacement measured by a fixed displacement transducer during one shaft rotation. Given a rotating element with negligible shaft bow or preset, the phase angle is a useful tool in determining unbalance orientation (direction), critical speed locations, and the amplification factors associated with the critical speeds. When the rotor operates at speeds below the fundamental critical speed, the maximum shaft displacement is nearly in phase with the rotor's unbalance. When the rotor operates at speeds above the first critical (but below the second critical) the phase angle of the maximum shaft displacement is opposed to (180 degrees out of phase from) the rotor's unbalance. During operation in the vicinity of the first critical speed, the phase angle changes rapidly from the low

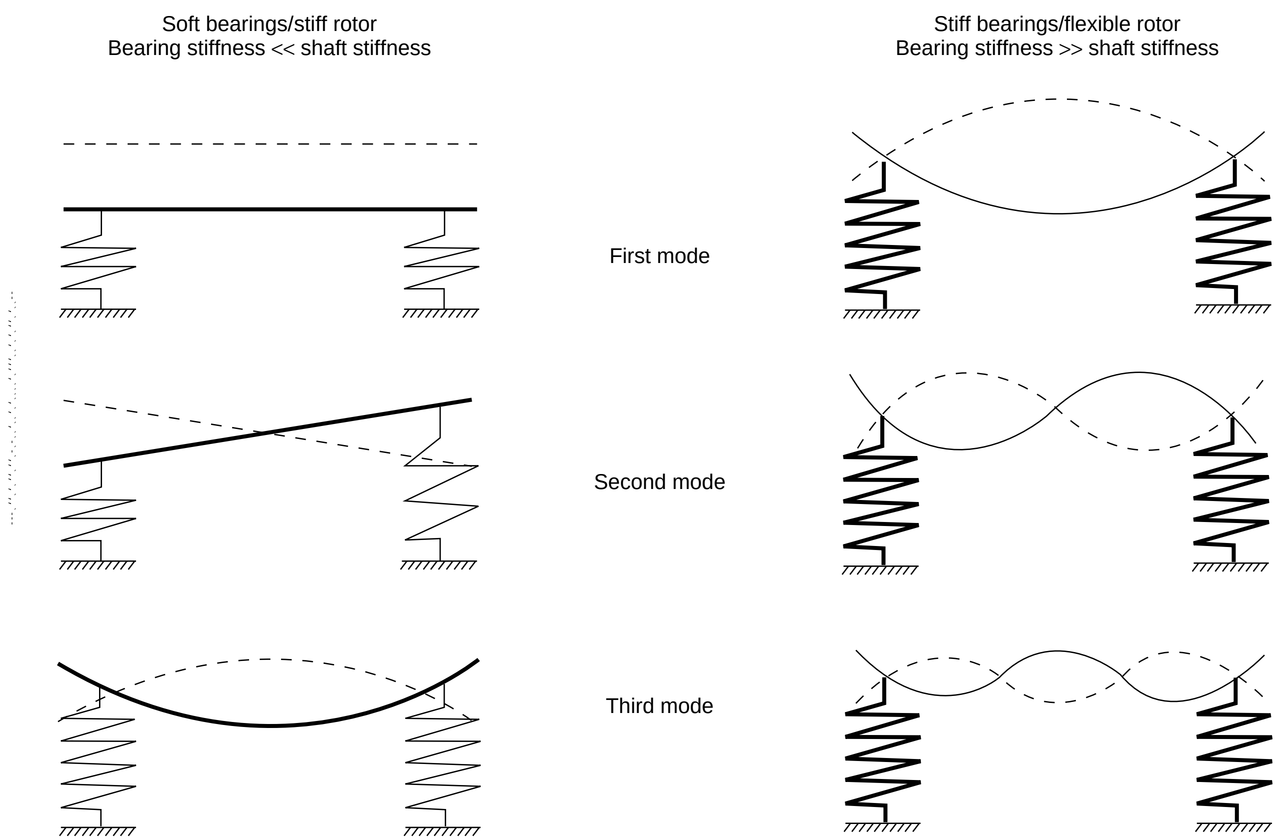


Figure 1-6—Mode Shape Examples for Soft and Stiff Bearings (Relative to Shaft Bending Stiffness)

speed value to the higher speed value. The rate at which the phase angle changes is related to the amplification factor associated with the critical speed.

1.2.2.14 *Resonance* is described by API (SP 2.8.1.1) as the manner in which a rotor vibrates when the frequency of a harmonic (periodic) forcing function coincides with a natural frequency of the rotor system. When a rotor system operates in a state of resonance, the forced vibrations resulting from a given exciting mechanism (such as unbalance) are amplified according to the level of damping present in the system. A resonance is typically identified by a substantial vibration amplitude increase and a shift in phase angle.

1.2.2.15 *Sensitivity to unbalance* is a measure of the vibration amplitude per unit unbalance, for example, micrometers per gram-millimeters (mils per ounce-inch).

1.2.2.16 *Stability* is a term referring to a unit's susceptibility to vibration at subsynchronous frequencies due to cross-coupled/destabilizing forces produced by stationary critical clearance components (such as bearings and seals) and rotating shrunk-on parts (such as impellers and shaft sleeves). Before the advent of tilting pad bearings, the principal cause of rotor instability was the cross-coupling generated by sleeve bearings, hence the natural association of the phenomenon with bearings only (*oil whirl*).

1.2.2.17 *Stiffness* is the equivalent spring rate in Newtons/millimeters (pounds/inches) of various elastic system elements. The rotor, bearings, supports, and so forth, have a characteristic stiffness each of which influences system lateral dynamics.

1.2.2.18 *Unbalance (imbalance)* is a measure that quantifies how much the rotor mass centerline is displaced from the centerline of rotation (geometric centerline) resulting from an unequal radial mass distribution on a rotor system. Unbalance is usually given in either gram-millimeters or ounce-inches.

1.2.3 FUNDAMENTAL CONCEPTS OF ROTATING EQUIPMENT VIBRATIONS

In order to understand the results of a rotor dynamics design analysis, it is necessary to first gain an appreciation for the physical behavior of vibratory systems. Begin by noting that all real physical systems/structures (such as buildings, bridges, and trusses) possess natural frequencies. Just as a tuning fork has a specific frequency at which it will vibrate when struck, a rotor has specific frequencies at which it will tend to vibrate during operation. Each resonance is essentially comprised of two associated quantities: the frequency of the resonance and the associated deflections of the structure during vibration at the resonance frequency. Resonances are often called *modes of vibration* or *modes of motion,* and the structural deformation associated with a resonance is termed a *mode shape*.

The modes of vibration are important only if there is a source of energy to excite them, like a blow to a tuning fork. The natural frequencies of rotating systems are particularly important because all rotating elements possess finite amounts of unbalance that excite the rotor at the shaft rotation frequency (synchronous frequency). When the synchronous rotor frequency equals the frequency of a rotor natural frequency, the system operates in a state of resonance, and the rotor's response is amplified if the resonance is not critically damped. The unbalance forces in a rotating system can also excite the natural frequencies of non-rotating elements, including bearing housings, supports, foundations, piping, and the like.

Although unbalance is the excitation mechanism of greatest concern in a lateral analysis, unbalance is only one of many possible lateral loading mechanisms. Lateral forces can be applied to rotors by the following sources: impeller aerodynamic loadings, misaligned couplings and bearings, rubs between rotating and stationary components, and so on. A detailed list of rotor excitation mechanisms is found in paragraph 2.8.1.5 of the API *Standard Paragraphs*. In-depth discussion of these is beyond the scope of this tutorial except for mentioning that applied lateral forces can be divided into two groups: steady and oscillatory. The steady loads (for example, steam loads, gear loads, and so forth) change the lateral system dynamic characteristics by altering static bearing loads. Oscillatory loads are further divided into two categories: harmonic and non-harmonic. Harmonic forces generate rotor responses only at the frequency of the applied load. The most common example of a harmonic lateral loading is unbalance. Non-harmonic forces may possess many frequency components. For example, rubs may excite rotor motion at the synchronous frequency as well as at fractional (½, ⅓, ¼, ...) and integer (2, 3, 4, ...) multiples of the synchronous frequency.

Although API (SP 2.8.1.5) notes that all oscillatory excitation mechanisms should be considered in the design of rotating equipment, the lateral analysis required by API (SP 2.8.1.5) is restricted to unbalanced forces. According to API (SP 2.8.1.5), the primary purpose of the lateral analysis is to calculate the frequency locations of a unit's lateral critical speeds and the unit's response sensitivity at the critical speeds to anticipated levels of unbalance. If the critical speeds are adequately separated from the unit's operating speeds or are heavily/critically damped, then the possibility of the unit encountering problematic vibrations from any excitation mechanism (including rubs) during normal operation is greatly reduced.

The vibration behavior of a rotor can be quantified with the aid of a simple physical model. Assume that a rotor-bearing system is analogous to the simple mass-spring-damper system presented in Figure 1-7. From physics, the governing equation of motion for this system can be written as Equation 1-1:

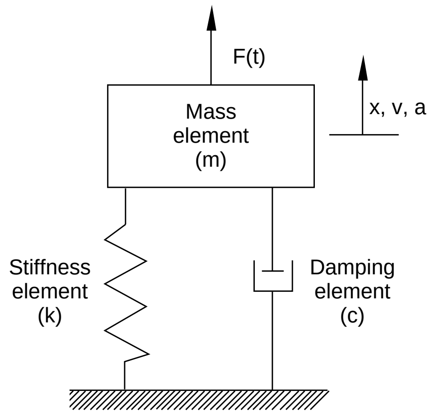


Table of Typical Units and Conversions

Quantity	SI Units (MLT System of Units)	US Customary Units (FLT System of Units)	US-to-SI Conversion[1]
g (gravitational acceleration)	9.88 m/s^2	386.4 in/s^2	- NA -
M (mass) [=Weight/g]	kg	lbf•s^2/in (derived unit)	175.13
c (damping)	N•s/mm	lbf•s/in	0.17513
k (stiffness)	N/mm	lbf/in	0.17513
F (force)	N (derived unit)	lbf	4.4482
x (displacement)	µm	mils	25.4
v (velocity)	mm/s	in/s	25.4
a (acceleration)	g	g	1.0

Notes:
1. Multiply the quantities listed above (in US Customary Units shown) by the US to SI conversion factors to obtain the quantity in the SI units listed in the table.

2. NA = not applicable.

3. **Common Units**
s = seconds
g = acceleration

U.S. Customary Units
in. = inches
mil = 1.0 x 10^{-3} inches
lbf = pound (force)

SI Units
mm = millimeters
µm = micrometers
kg = kilograms
N = Newtons = kg•m/s^2

Figure 1-7—Simple Mass-Spring-Damper System

$$m\ddot{x}+c\dot{x}+kx=F(t) \tag{1-1}$$

Where:

- m = mass of the block.
- c = viscous damping coefficient.
- k = stiffness of the elastic element.
- x = displacement of the block.
- $F(t)$ = force applied to the block (time-dependent function).

In this example, the displacement response of the block to the applied force is counteracted by the block's mass and the support's stiffness and damping characteristics. The undamped natural frequency of this system is calculated by determining the eigenvalue of the second order homogeneous ordinary differential equation ($F = 0$) for the case where the damping term is neglected ($C = 0$) as seen in Equation 1-2.

$$\omega=\sqrt{\frac{k}{m}} \tag{1-2}$$

Where:

- ω = undamped natural frequency.

The damped natural frequency of the homogeneous system ($F = 0$) is explicitly evaluated below.

$$\begin{aligned}\lambda &= p + i\cdot w_d\\ p &= \frac{c}{2m}\\ \omega_d &= \sqrt{\frac{k}{m}-\left(\frac{c}{2m}\right)^2}\end{aligned} \tag{1-3}$$

Where:

- p = damping exponent
- w_d = frequency of oscillate

Note that the oscillatory frequency of the damped system, ω_d, is equal to the undamped natural frequency of the system only when system damping is negligible. In a practical sense, this occurs in turbomachinery only when the mode shape indicates that the journal motion in the bearing is less than 5 percent of the shaft midspan displacement. This observation underscores the fact that an undamped critical speed analysis should, in general, not be used to define the critical speeds of a rotating machine.

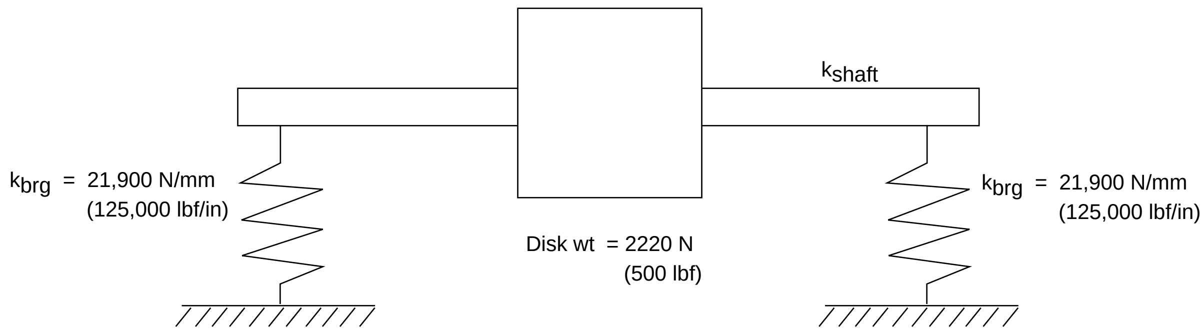
While the single degree of freedom system examined above is useful for examining the general concepts of vibration theory, this system is clearly not representative of a turbomachine. An accurate model of a rotating assembly is comprised of many small blocks or lumped masses that are connected together by a network of elastic springs. Sophisticated mathematical techniques such as the Finite Element Method or the Transfer Matrix (Myklestad/Prohl) Method are typically used to systematically generate the set of equations that describe the dynamic behavior of the rotating assembly. Once the mathematical model of the rotating element is generated, it is connected to ground through linearized elastic stiffness and viscous damping elements that represent the fluid film support bearings.

A simple undamped system, similar in appearance to a beam rotating machine, is presented in Figure 1-8. This system is comprised of a massive, rigid disk that is held between two identical elastic bearings/supports by a massless shaft. If the shaft is assumed rigid or extremely stiff relative to the bearings/supports, then the primary sources of flexibility in the system are the two bearing/support systems. If the weight of the disk is 2224.1 Newtons (500 pound-force) and the bearing/support has a stiffness of 21,891.3 Newtons/millimeter (125,000 pound-force/inch), then the natural frequency of the system is approximately 4200 cpm. In reality, the shaft supporting the disk will also possess flexibility. In fact, it is not uncommon in centrifugal compressors for the shaft to be significantly more flexible than the bearings. To examine the effect of shaft flexibility on the vibration characteristics of this simple system, let k_{shaft} = 8756.5 Newtons/millimeters (50,000 pound-force/inch) or approximately 20 percent of the bearing stiffness. The two elastic elements supporting the single disk (shaft and bearings/supports) may be replaced by a single equivalent spring that is connected to ground. The stiffness of this single spring is a function of the stiffness of the shaft and the two bearings. This stiffness is calculated using Equation 1-4 as follows:

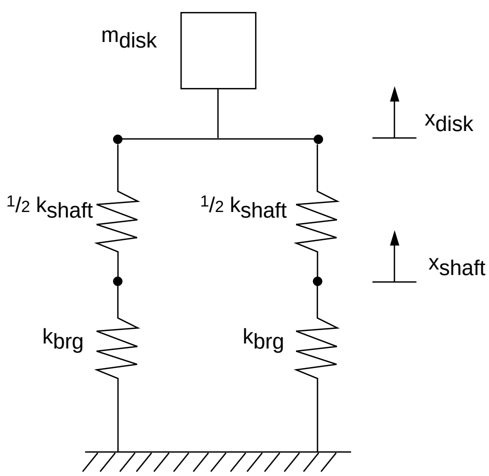
$$\frac{1}{k_{equivalent}}=\frac{1}{k_{shaft}}+\frac{1}{k_{bear}} \tag{1-4}$$

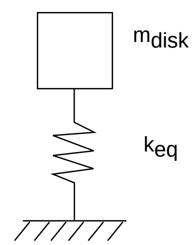
This equation indicates that the stiffness of the combined shaft-bearing system will be less than the stiffness of the single most flexible element. In this example, the shaft is the single most flexible element (k_{shaft} = 8756.5 Newtons/millimeter or 50,000 pound-force/inch). According to Equation 1-3, the effective stiffness of the combined shaft-bearing spring system is only 7297.7 Newtons/millimeter (41,670 pound-force/inch) and the calculated natural frequency of the system with the flexible shaft will decrease by more than 59 percent from the original value of 4200 cpm to 1710 cpm.

Although the discussion above highlights the potential effect of shaft flexibility on the location of a shaft's fundamental natural frequency, this discussion does not illustrate why shaft flexibility is detrimental to the lateral dynamic characteristics of a rotating machine. To examine this question, consider the dynamic system displayed in Figure 1-9. Note that this system is identical to the system just discussed except that viscous damping elements have been added to the bearing model. All oil film bearings generate significant viscous damping forces. Such bearings support virtually all critical petroleum plant process rotating equipment. Figure 1-10 displays the calculated response of the disk to a harmonic load acting at the disk for various values of shaft stiffness. Note that as the shaft stiffness decreases, the peak response



A SIMPLIFIED MODEL OF A BEAM-TYPE ROTATING MACHINE





$$k_{eq} = 2\left\{\frac{1}{\dfrac{2}{k_{shaft}} + \dfrac{1}{k_{brg}}}\right\}$$

DYNAMIC SYSTEM SCHEMATIC

EQUIVALENT DYNAMIC SYSTEM

k_{shaft} (N/mm)	k_{shaft} (lbf/in)	Natural Frequency (CPM)
∞	∞	4197
8750	50.000	1714

Figure 1-8—Effect of Shaft Bending Stiffness on Calculated Natural Frequencies (Simplified Model of a Beam-Type Machine)

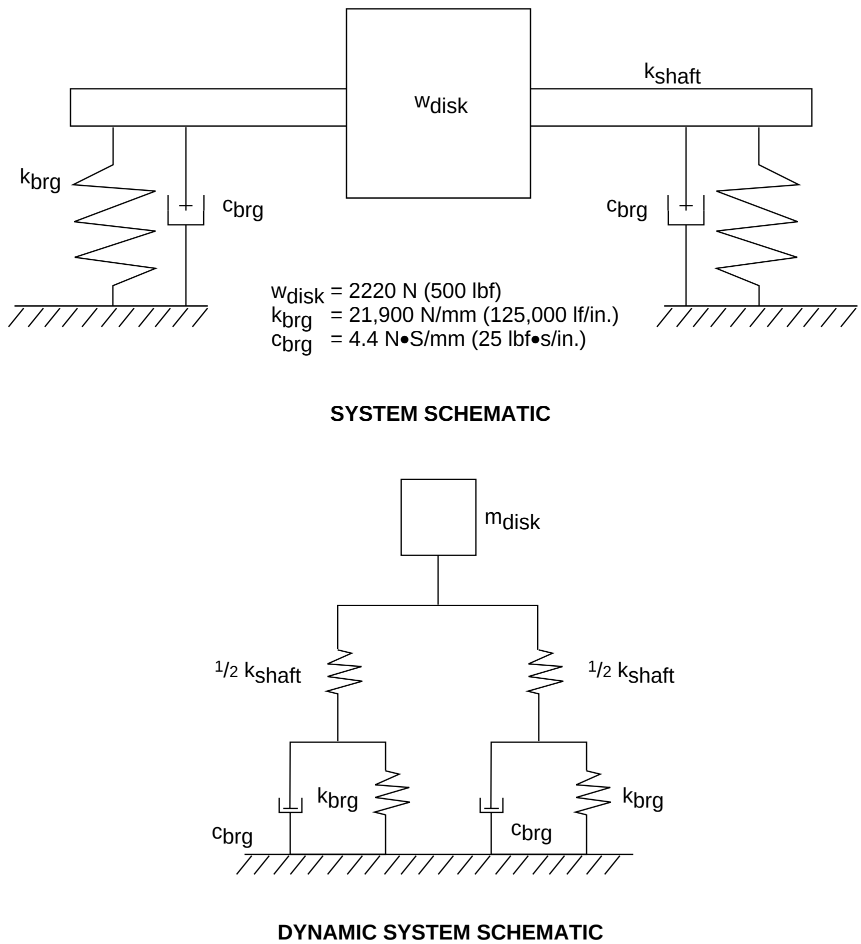
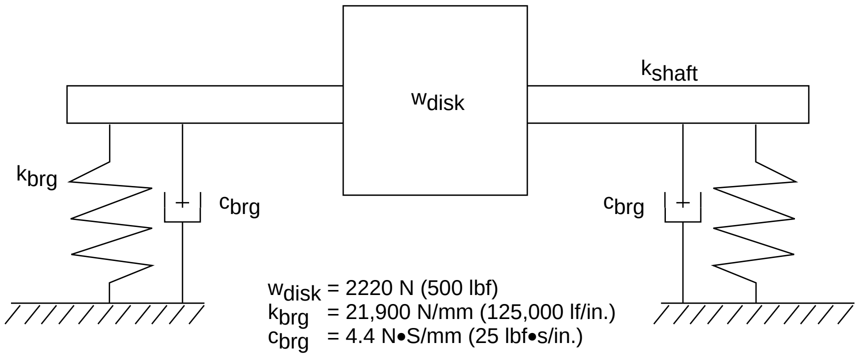


Figure 1-9—Simple Rotor-Support System With Viscous Damping

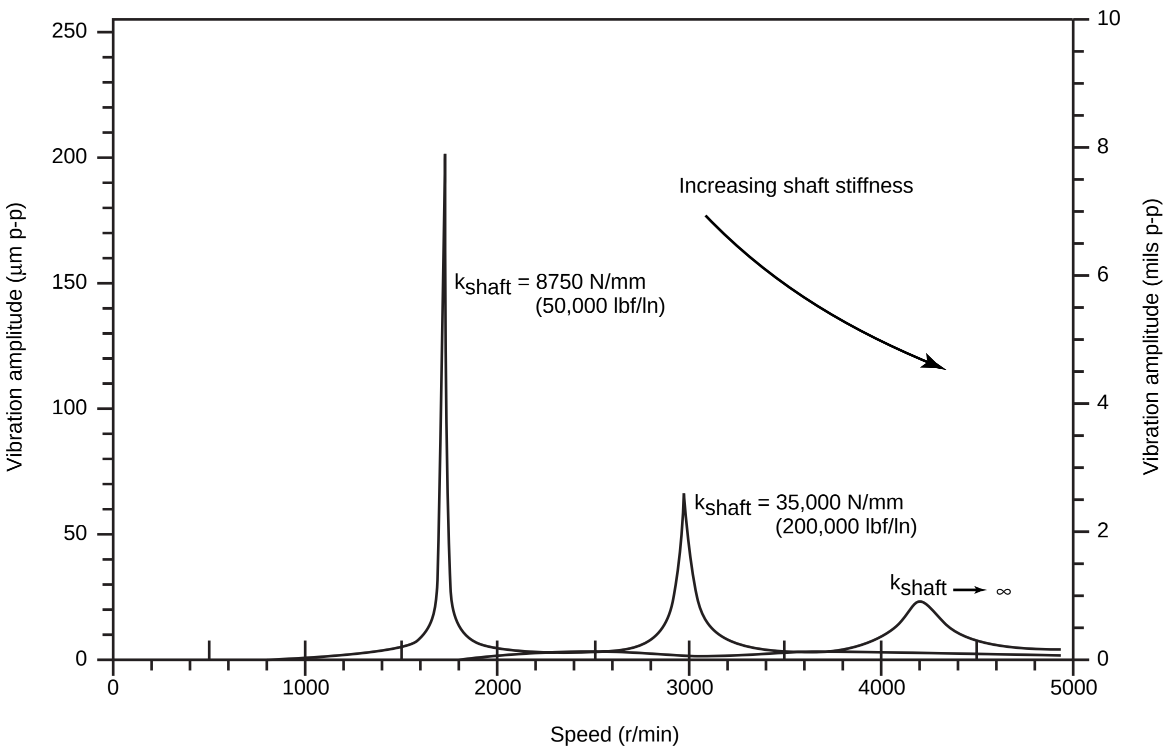
frequency decreases while the amplitude of the peak response and the *sharpness* of the peak both increase. These observations are understood by noting that the decrease in shaft stiffness decreases the relative deflection of the shaft in the bearings and diminishes the magnitude of the damping forces provided by the bearings. Thus, one may conclude that the effect of damping provided by the bearings is maximized when the shaft stiffness is large relative to the bearing stiffness.

This example permits the development of a general classification of well-designed rotating equipment. Experience indicates that a rotor-to-bearing stiffness ratio should be greater than 0.25. Machines with stiffness ratios lower than this value tend to be prone to vibration problems resulting from the excessive rotor flexibility, such as highly-amplified critical speeds. Note that flexible shaft machines are also more prone to rotor dynamic stability problems.

The discussion regarding the effect of shaft stiffness on the lateral dynamic characteristics of a simple rotating assembly highlights the need for well-behaved rotating equipment to possess adequate shaft stiffness relative to the stiffness of the bearings. The presence of adequate shaft stiffness permits the bearings to dampen the rotor's vibrations caused by unbalance. By definition, damping is a dissipative force that converts mechanical energy in the vibrating shaft into heat that is transported away from the shaft by lubricant flowing out of the bearing housings. Although bearing damping affects rotor response to unbalance at all operating speeds, the effect of bearing damping is most pronounced at the rotor's critical speeds. Figure 1-11 displays the influence of bearing damping on a single mass rotor's response to a midspan unbalance through its critical speed. Examination of this figure reveals that bearing damping influences the location of the critical speed and controls the amplitude of the response near the critical speed. As bearing damping is increased, both the rotor's peak response amplitude and the associated amplification factor decrease. This figure indicates that without bearing damping, the rotor's amplitude of vibration at the critical speed would be extremely high and would likely prevent safe supercritical operation.



SYSTEM SCHEMATIC



CALCULATED RESPONSES FOR SEVERAL VALUES OF SHAFT STIFFNESS

Figure 1-10—Effect of Shaft Stiffness on the Calculated Critical Speed and Associated Vibration Amplitude For a Simple Rotor-Support System with Viscous Damping

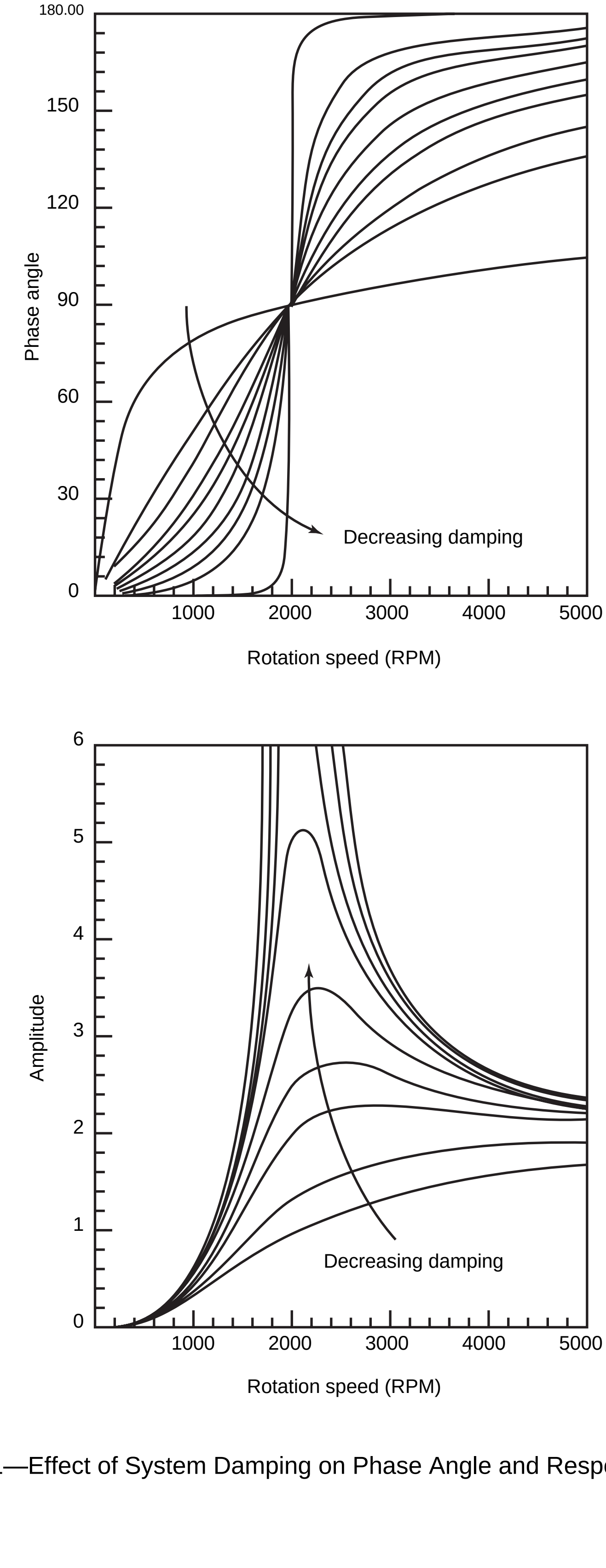


Figure 1-11—Effect of System Damping on Phase Angle and Response Amplitude

It is hoped that the discussion above provides the reader with an appreciation of relationship between shaft stiffness and the bearing stiffness. Adequate shaft stiffness is a prerequisite for reliable turbomachine operation because an extremely flexible shaft robs the bearings of their ability to dampen vibrations. Given a shaft that possesses adequate bending stiffness, the fluid film bearings must be designed to provide the right amount of stiffness and damping to *tune* critical speeds away from the operating speed range and minimize the associated amplification factors.

1.2.4 ELEMENTS OF A STANDARD ROTOR DYNAMICS ANALYSIS

The purpose of a *standard* rotor dynamics analysis and design audit is to enable an engineer to characterize the lateral dynamics design characteristics of a given design. One might compare such analysis to the routine physical in which a doctor seeks to determine a patient's general health rather than specifically testing for the source of a known problem or developing a comprehensive treatment plan for a specific disease. While analysis of some rotating equipment may require analysis specific to the unit, a general method has emerged for performing the standard lateral analysis. The standard analysis is composed of four parts: (a) rotor-bearing system modeling, (b) undamped critical speed analysis, (c) damped unbalance response analysis, and (d) rotor dynamic stability (damped eigenvalue) analysis.

1.3 References

The following standards contain provisions that, through reference in this text, constitute provisions of this standard:

API

- Std 541 F*orm-Wound Squirrel Cage Induction Motors—250 Horsepower and Larger*
- Std 613 *Special Purpose Gear Units for Petroleum, Chemical, and Gas Industry Services*
- Std 617 *Centrifugal Compressors for Petroleum, Chemical, and Gas Service Industries*
- Std 670 *Vibration, Axial-Position, and Bearing-Temperature Monitoring Systems*

1.4 Rotor Bearing System Modeling

1.4.1 GENERAL

Modeling is the single most important process in performing any engineering analysis of a physical system. Several checks should be incorporated into the modeling procedure in order to assure the designer that the model accurately simulates the dynamic behavior of the design. If the model does not accurately simulate the proposed design, the sophisticated analysis and evaluation of the design will do little good. The five steps taken to model rotating equipment are listed in sequence below:

a. Generate a mass-elastic lateral model of the unit's rotating assembly.
b. Calculate static bearing reactions (including miscellaneous static load mechanisms such as gear loading, loads resulting from partial arc steam admission, and so forth).
c. Calculate linearized fluid film bearing coefficients.
d. Calculate linearized floating ring oil seal coefficients (if present).
e. Calculate all other excitation mechanisms (such as aerodynamic effects and labyrinth seal effects).

These steps will be discussed in greater detail in the following pages.

1.4.2 UNDAMPED CRITICAL SPEED ANALYSIS

The undamped critical speed analysis is usually performed as a precursor to the response analysis. The primary result of the undamped analysis is the undamped critical speed map, which graphically displays the effect of total bearing/support system stiffness on the rotor's undamped critical speeds. A typical critical speed map is presented in Figure 1-5. This plot typically presents the first three undamped, forward-whirling modes as a function of total bearing/support stiffness. Speed-dependent total bearing/support principal stiffnesses (k_{xx}, k_{yy}) are often cross-plotted to help identify the unit's critical speeds. Both the abscissa (bearing stiffness) and ordinate (frequency of the critical speed) axes of the critical speed map are generally log scales.

Note that in beam-type machines (no overhung wheels or stages), the difference in bearing static loads often does not vary significantly from end to end. Consequently, the calculated bearing coefficients often are nearly equal on both ends of the unit. For this reason, only one set of a bearing's coefficients typically appears on a critical speed map. If the difference in the journal static loads is greater than 20 percent of the total rotor weight, then two separate critical speed maps should be generated with each bearing's principal stiffnesses cross-plotted as functions of rotor speed.

As previously mentioned, the undamped critical speed analysis should not be used to determine critical speeds and associated separation margins. This analysis does not include a variety of effects such as damping generated by bearings and seals that significantly impact the location of critical speeds as defined by API in Standard Paragraph 2.8.1.3 (see also Figure 1-1). As discussed in the following, an undamped critical speed map can be of great value in rapidly determining the general dynamic characteristics of rotating equipment. According to API Standard Paragraph 2.8.2.4.e, however, the undamped critical speed analysis is performed by the equipment manufacturer only at the request of the customer.

It was noted in the preceding that shaft flexibility can substantially decrease a rotor-bearing system's lateral critical speeds and otherwise degrade the lateral rotor dynamics characteristics of a given piece of rotating equipment. In reality, however, the dynamic characteristics of a turbomachine are degraded not by the mere presence of shaft flexibility, but rather by *excessive* shaft flexibility compared with the flexibility of the support bearings. This can be understood by noting that when the stiffness of the bearings are small relative to the bending stiffness of the rotating element, then the frequency of the unit's fundamental critical speed is principally governed by the stiffness of the bearings and the mass of the rotor. Conversely, when the bearings are much stiffer than the bending stiffness of the shaft, then the frequency of the unit's undamped critical speeds will principally be governed by the mass and bending stiffness of the rotor. The former situation (stiff rotor/soft bearings) is desirable from a dynamics standpoint while the latter (flexible rotor/stiff bearings) is not. The undamped critical speed map provides a useful tool for determining when shaft flexibility is excessive compared with the anticipated or calculated range of bearing stiffness and may compromise the safe and reliable operation of a turbomachine.

In the left side of the undamped critical speed map displayed in Figure 1-5 lies a region of bearing stiffness where the slope of the two fundamental (lowest frequency) critical speed lines is positive and approximately constant. This area is called the *stiff rotor* part of the critical speed map because the bending stiffness of the rotor is appreciably greater than the bearing stiffness. In the right hand side of the undamped critical speed maps lies an area where the critical speeds do not change with increasing bearing stiffness. This area is referred to as the *stiff bearing* part of the critical speed map because the bearing stiffness is significantly greater than the bending stiffness of the rotor. The asymptotic values of the critical speed lines are often referred to as the *rigid bearing critical speeds*.

Cross-plotting the calculated bearing coefficients on undamped critical speed maps allows one to infer the general damped unbalance response characteristics of a rotor-bearing system. If the bearing stiffness intersects a critical speed line in the *stiff rotor* part of the undamped critical speed map, then the amplification factor associated with the critical will be small (probably be less than 5.0), and the rotor's response to unbalance during operation near the critical speed will be well-damped. The cross-plotted bearing coefficients displayed in Figure 1-5 intersect the two fundamental critical speed lines in the stiff rotor part of the critical speed map. If, however, the bearing/support stiffnesses intersect a critical speed line on the flat part of the critical speed line, then the bearings are much stiffer than the rotor, and the amplification factor associated with the critical speed will be large (probably greater than 12.0). The rotor's response to unbalance will also be highly amplified during operation near the critical speed.

The relationship between the undamped critical speed map and the results of other lateral dynamics analysis, such as the damped unbalance response analysis, may be better understood if the undamped mode shapes associated with the first three undamped critical speeds are examined for the *soft* and *stiff* bearing cases (Figure 1-6). Note in the case of *soft* bearings (relative to shaft bending stiffness) that shaft deflections are small relative to the bearing deflections. The damping generated by the bearings will be used to attenuate rotor vibrations caused by potential rotor exciting forces such as unbalance. On the other hand, when the bearings are *stiff* relative to the shaft bending stiffness, the shaft deflections are large relative to the bearing deflections. In this case, even if the bearing damping coefficients are large, the damping forces provided by the bearings will be small because the rotor motion at the bearings is small. Thus, rotor vibrations caused by unbalance and other forces will be highly amplified at critical speeds if the bearings are much stiffer than the rotor bending stiffness.

Mode shapes associated with the undamped lateral natural frequencies can be calculated as a byproduct of the undamped critical speed analysis. Mode shapes are usually calculated using bearing principal stiffnesses evaluated at the unit's normal operating speed. Consequently, the calculated lateral natural frequencies do not exactly match the unit's critical speeds defined by the intersection of the critical speed lines with the bearing/support principal stiffnesses. These plots display the rotor's normalized free (unforced) rotor deflections associated with the lateral undamped natural frequencies. The undamped mode shapes are useful for the following reasons:

a. The undamped mode shapes are *planar* or two dimensional; undamped mode shapes do not possess the two dimensional bending displayed by the rotor during the damped response analysis.

b. The undamped mode shapes are not dependent upon an unbalance distribution and are characteristic of the mass-elastic model. Furthermore, the undamped mode shapes associated with the first three critical speeds always possess certain basic shapes for beam rotors. For example, the mode shape associated with the fundamental critical always possesses rigid body translational and rotational motion with some shaft bending. The third critical/*first bending* mode possesses shaft bending only. The characteristic shapes of the mode deflections associated with the fundamental three criticals permit the critical speed calculations to be verified. The complete set of the three lowest frequency undamped lateral natural frequencies then may serve as a reference against which the damped lateral response and damped rotor stability calculations can be checked.

c. The undamped mode shapes provide a good indication of the relative displacements that the shaft undergoes when the rotor operates in the vicinity of the associated critical speed. Thus, given a vibration amplitude at a probe location during operation near a critical speed, one may estimate the vibration amplitude at other locations on the rotating element.

1.4.3 DAMPED UNBALANCED RESPONSE ANALYSIS

The damped unbalance response analysis is the principal tool used by API to evaluate relevant lateral rotor dynamics characteristics including lateral critical speeds and associated amplification factors. Figure 1-1 reproduces the sample rotor response plot from the API *Standard Paragraphs*. Note that critical speeds, amplification factors, and critical speed separation margins are all defined using information presented in calculated or measured rotor response plots.

Accuracy of calculated results is obviously dependent upon the level of detail incorporated in the rotor system model. The model used for the response analysis must incorporate a variety of effects, which will be discussed in the following sections. Once the model of the unit is constructed, the vendor calculates the lateral response to known amounts of unbalance applied to specific locations on the rotor. The locations of unbalance application are prescribed by API in the standard paragraphs to ensure that the specific *criticals of concern* are excited.

As required by the API *Standard Paragraphs*, the results of the damped unbalance response analysis will include the following items:

a. Bodé response plots: Bodé plots display the calculated amplitude and phase of the vibration resulting from applied unbalance as a function of the operating speed of the rotor. These plots are normally provided, for several axial shaft locations: shaft displacement probe locations, bearing locations, rotor midspan, and so forth.

b. Critical speed separation margins: The critical speed separation margins indicate the proximity of the rotor's calculated critical speeds to the machine's operating speed or speed range. The critical speed separation margins are expressed as the per cent of minimum and maximum operating speeds that a critical speed is removed from the operating speed range.

c.Amplification factors associated with critical speeds: The amplification factor is a non-dimensional value that indicates the sensitivity of the rotor to unbalance at a critical speed. *Amplification factor* is defined in the sample response plot displayed in Figure 1-1. Some vendors also provide Rotor Unbalance Sensitivities, S_i, for each critical speed. The S_i are defined as follows in Equation 1-5:

$$S_i = \frac{A_{ci}}{U} \tag{1-5}$$

Where:

A_{ci} = Response amplitude at ith critical.
U = magnitude of the applied balance.

d. Mode shapes: Dynamic rotor mode shapes or modal diagrams display the major axis amplitude of the response at critical clearance or otherwise important locations such as coupling engagement planes, bearings, and seals.

1.4.4 ROTOR DYNAMIC (DAMPED EIGENVALUE) STABILITY

Most purchasers of centrifugal compressors recognize the importance of determining the rotor dynamic stability of a unit prior to construction by requiring that a stability or damped eigenvalue analysis be accomplished as part of the basic design audit. The goal of this analysis is to determine if a unit is susceptible to large amplitude subsynchronous vibration during normal operation. If the proposed unit design is not sufficiently stable, then modifications to the bearing and/or shaft design should be accomplished prior to unit construction. Field solution of a compressor rotor stability problem can be extremely vexing because the engineer can rarely make significant improvements in the unit's stability once the rotor is built without lowering unit speed and flow capacity. The engineer generally attempts to control an unstable rotor by adjusting bearing and seal lubricant supply temperatures, reducing inlet pressure, and so forth. Even if these measures attenuate the unstable rotor vibrations, the unit is likely to be problematic until it is redesigned.

Unfortunately, this analysis requires much input that is difficult to accurately predict: stage aerodynamic interactions with the casing, labyrinth seal effects, friction effects from shrunk-on components, and so forth. For this reason, almost all stability analysis must be calibrated by experience according to unit service and general design characteristics. For example, high-pressure centrifugal compressors have displayed significant subsynchronous vibration, even though calculations indicated that the basic designs were well-conceived. Rotor stability is most influenced by the relative shaft-to-bearing stiffness; the design of the rotor, bearings, and oil seals; and the level of destabilizing forces caused by process conditions.

Current technology identifies the damped eigenvalues, evaluated at the rotor's operating speed, as the principal measure of rotor stability. The damped eigenvalue is a complex number of the form $s=p +/- i\omega_d$ where p is the damping exponent, ω_d is the frequency of oscillation, and i = *square root (-1)*. The effect of the sign of the damping exponent on the motion of the rotor is presented in Figures 1-12 and 1-13. As previously noted, if the damping exponent of an eigenvalue is negative, then the rotor vibrations associated with this mode will be stable (envelope of vibrations decreases with time). If the damping exponent of an

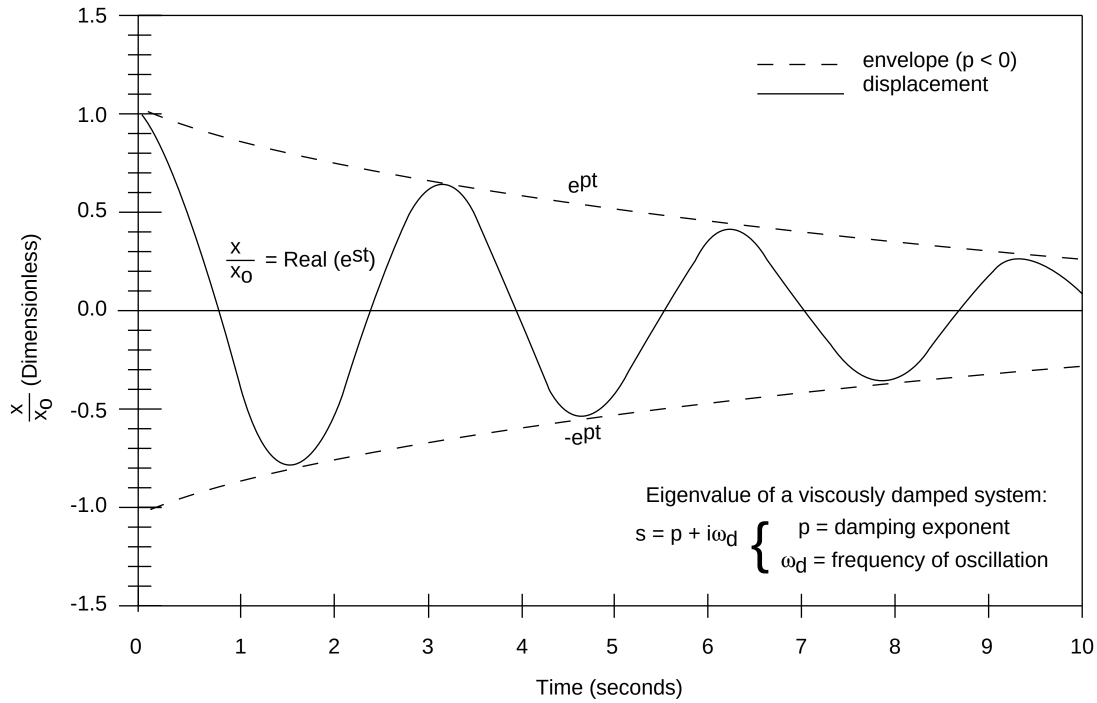


Figure 1-12—Motion of a Stable System Undergoing Free Oscillations

eigenvalue is positive, then the rotor vibrations associated with the mode will be unstable (envelope of vibrations increases with time).

Although the real part of the complex eigenvalue is the direct result of rotor stability calculations, many engineers evaluate rotor stability using a derived quantity called *log decrement* (see 1.2.2.9). The log decrement, δ, is calculated as follows:

$$\delta = -\frac{2\pi\omega_d}{p} \tag{1-6}$$

The log decrement is a measure of how quickly the free vibrations are experienced by the rotor system decay. When the log decrement is positive, the system is stable. Conversely, when the log decrement is negative, the system is unstable. The log decrement has proved to be a useful measure of rotor stability because it is a non-dimensional quantity and may be interpreted using general design rules.

The uncertainty surrounding the rotor stability analysis and the attendant lack of clear-cut guidelines for evaluating a design probably account for the absence of the topic from the API *Standard Paragraphs*. Experience with stability problems, however, clearly indicates that a rotor stability analysis should be conducted for all centrifugal compressors with interpretation of results and suitability of design to be mutually agreed upon by the vendor and purchaser.

1.5 Modeling Methods and Considerations

1.5.1 GENERAL

It has become axiomatic to engineers wishing to perform an accurate computer simulation of physical systems that only accurate models beget accurate results. This section will describe some of the methods that have been successfully employed by the authors over a period of many years to model the important elements of a solid shaft rotor-bearing system. Although tie-bolt or built-up rotors can be accurately modeled using the techniques outlined below, the reader is cautioned that tie-bolt rotors are potentially subject to greater modeling complications than solid shaft rotor designs. For example, the rotor's bending stiffness characteristics may be related to the tie-bolt stretch. The non-linear axial face friction forces between rotor segments may become significant if the segments move relative to each other during rotor operation. Finally, small high-speed built-up rotors may simply not be adequately represented by direct application of cylindrical beam elements. In such cases, sophisticated finite element analysis of the rotating element may be necessary to build an equivalent beam element model that permits accurate prediction of results.

1.5.2 ROTOR MODELING

An accurate model of a rotor system is a model that permits accurate calculation of the actual rotor system's dy-

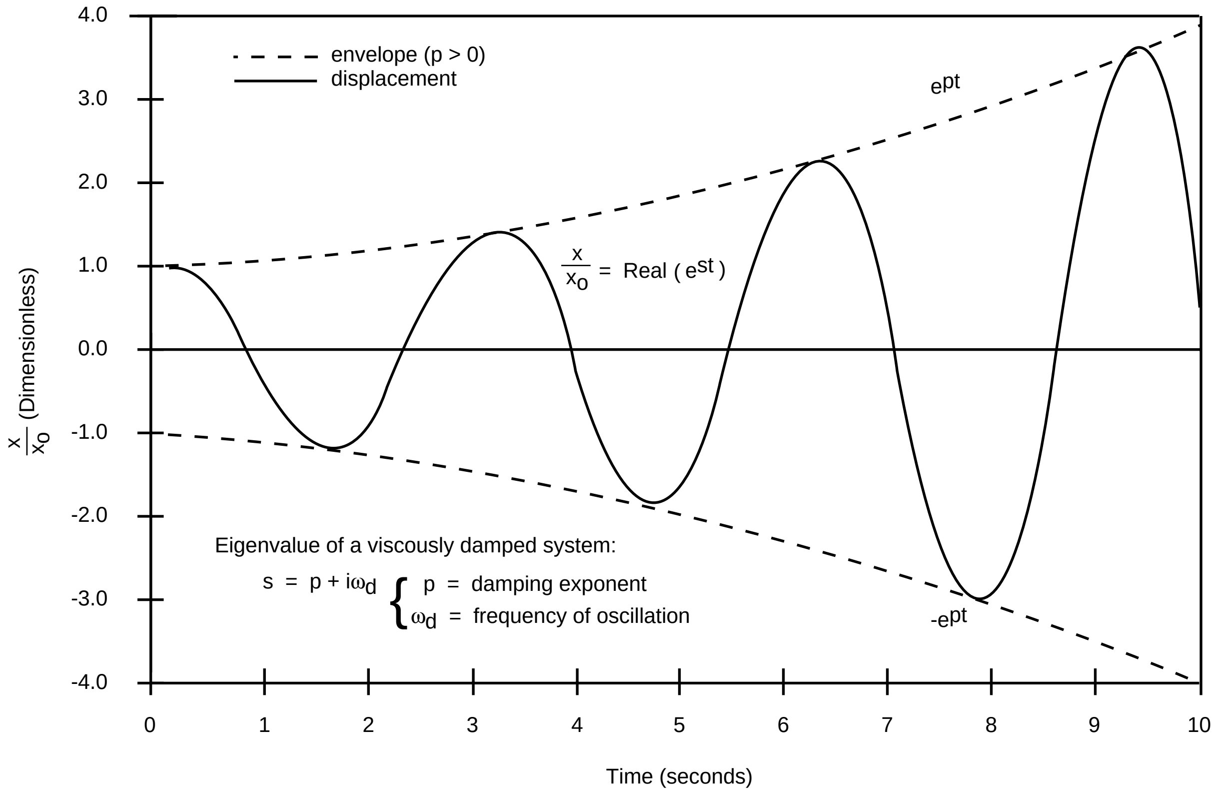


Figure 1-13—Motion of an Unstable System Undergoing Free Oscillations

namic characteristics. This occurs when the rotor's mass-elastic (inertia-stiffness) properties are adequately represented. For the purpose of performing a basic rotor dynamics design audit, two simple building blocks—lumped inertia shaft and disk elements—are joined together to form a complete model of the rotating assembly. Shaft elements contribute both inertia and stiffness to the global model; whereas, disk elements contribute inertia only. More complicated element types can be used at the cost of introducing complexity to the model. In general, however, most petroleum plant turbomachinery can be adequately modeled using the lumped inertia shaft and disk elements presented in this tutorial.

Once the type of elements to be used in the analysis has been established, it simply remains for the engineer to generate a description of the subject rotor's geometry using a sufficient number of the selected elements. Schematics of a rotor and its associated lumped parameter model are displayed in Figure 1-14. Some general constraints must be placed on the use of the lumped inertia shaft elements, however, to ensure that accurate rotor models emerge from the process. Clearly, if too few elements are used the resulting model may not possess sufficient resolution to accurately capture some of the detailed mass-elastic properties of the rotating assembly. If a large number of elements are used to model the rotor, then numerical problems may result. A secondary benefit of minimizing the number of elements used to produce a rotor model is a reduction of the amount of time needed by the engineer to generate data files and for the computer to perform calculations.

1.5.2.1 Division of Rotor into Discrete Sections

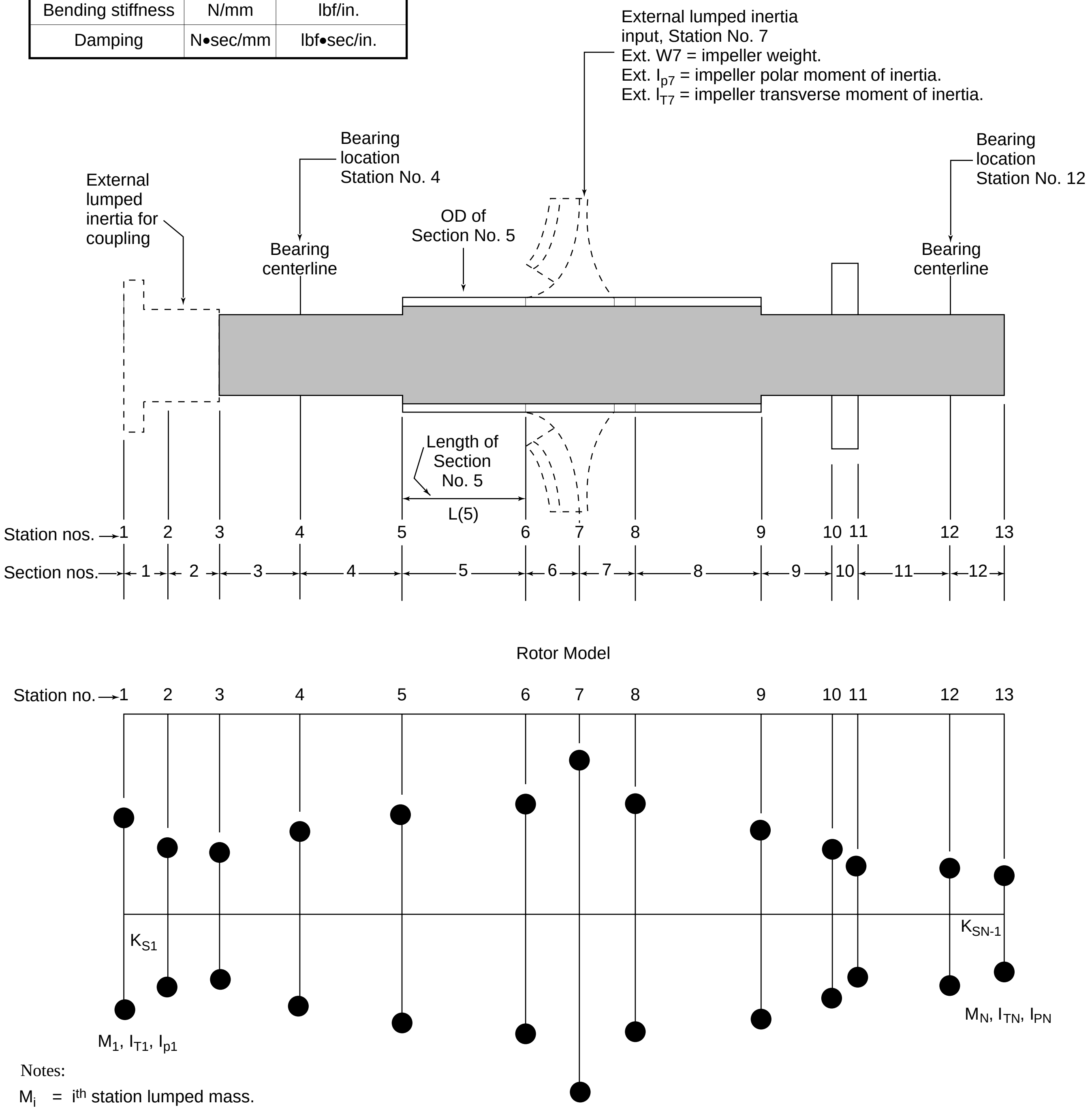
The modeling process starts with the analyst's dividing the rotor into a series of elements that begin and end at step changes in the outside diameter (OD) or inside diameter (ID). Once initial division of the rotor has been accomplished, further refinement of the model is almost certainly required. The following simple guidelines are proposed:

a. The length to diameter ratio of any section should not exceed 1.0 (0.5 is preferred).
b. The length to diameter ratio of any section should not be less than 0.10.

The first guideline is proposed to ensure that the model possesses sufficient resolution to permit accurate calculation of the first three critical speeds. The second guideline is proposed to ensure that large length changes in adjacent shaft elements are avoided as this practice may generate numerical calculation problems. When a large length difference exists between adjacent shaft elements, a large difference in the re-

Typical Units for Input

Parameter	SI Units	US Units
Length	mm	in.
Diameter	mm	in.
Weight	N	lbf
Moment of Inertia	N•mm^2	lbf•in.^{2}
Bending stiffness	N/mm	lbf/in.
Damping	N•sec/mm	lbf•sec/in.



Notes:

M_i = i^{th} station lumped mass.
I_{Ti} = i^{th} station lumped transverse moment of inertia.
I_{pi} = i^{th} station lumped polar moment of inertia.
K_{si} = i^{th} shaft stiffness section bending stiffness.

Figure 1-14—Schematic of a Lumped Parameter Rotor Model sulting shaft stiffness is created that will cause numerical round-off errors to accumulate when these stiffnesses are added together during the model assembly process.

Whenever the analyst is unsure of how to model a given feature in the rotating element, he or she may always proceed by determining the sensitivity of calculated results to various ways of modeling the feature in question. For example, if one strictly adheres to the two modeling guidelines proposed above, a short circumferential groove machined into the shaft cannot be modeled. Such grooves are often found on compressor shafts to locate split rings at the ends of the aerodynamic assembly and to lock thrust collars onto the shaft. The authors generally ignore such design features when analysis indicates that decreasing the diameter of the entire element encompassing the groove does not affect the criticals of concern or the associated modeshapes. When a given geometric feature possesses a strong influence on calculated results, the designer must examine the possibility that the rotor's design may be fundamentally flawed.

On those occasions when the analyst has difficulty modeling a rotating assembly because the rotor geometry cannot be readily described using rudimentary shaft elements, then an equivalent model can be formulated from more sophisticated analysis. For example, the bending characteristics of a *stub shaft* bolted to the second stage impeller on an overhung gas pipeline compressor have been determined using a finite element analysis of the piece. The finite element mesh is displayed in Figure 1-15. Note that the large counter-bored bolt holes dramatically decrease the stub shaft's lateral bending stiffness. Once the static bending analysis of the component is accomplished, an equivalent lumped parameter beam-type model of the type used in rotor dynamics analysis can be formulated that possesses identical bending stiffnesses at the lumped inertia locations.

1.5.2.2 Addition of External Masses and Inertial Loadings

Not all rotating assembly components contribute to the bending stiffness of the rotor. In fact, most components that are shrunk onto centrifugal compressor shafts (impellers, sleeves, thrust collars, and so on) are assumed not to affect the bending stiffness of the rotating element except in unusual cases. Although this assumption generally results in under-prediction of the fundamental critical speed, the difference between the actual and calculated critical speeds is usually less than 10 percent. The actual difference between the calculated and observed critical is dependent on the flexibility of the rotating element because the more flexible the shaft, the greater the effect of shrunk-on sleeves and impellers. This observation justifies use of the preceding assumption when performing a standard rotor dynamics analysis, as an extremely flexible rotor generally operates far above the first critical. Other important rotor dynamic characteristics of the rotor system such as amplification factor, rotor stability, and second critical location are all either less sensitive to the influence of shaft sleeves or are more conservatively predicted by neglecting the stiffening effect of the shaft sleeves. Note that the model used to predict the unit's critical speeds may have to be refined according to data collected during mechanical testing of the actual machine if the critical speeds and associated amplification factors differ by more than 5 percent.

Components shrunk onto the shaft do affect the inertial characteristics of the rotating assembly, however, and must be added to the model. This is most often accomplished by adding lumped inertias at the mass centers of the shrunk-on components. It is occasionally necessary, as in the case of motor cores, to generate detailed inertia distributions of the shrunk-on component. Most rotors will include at least several of the following additional masses:

a. Impellers/disks.
b. Couplings.
c. Sleeves.
d. Balance pistons.
e. Thrust collars.

Particular machines will have specific masses that must be added, including the following:

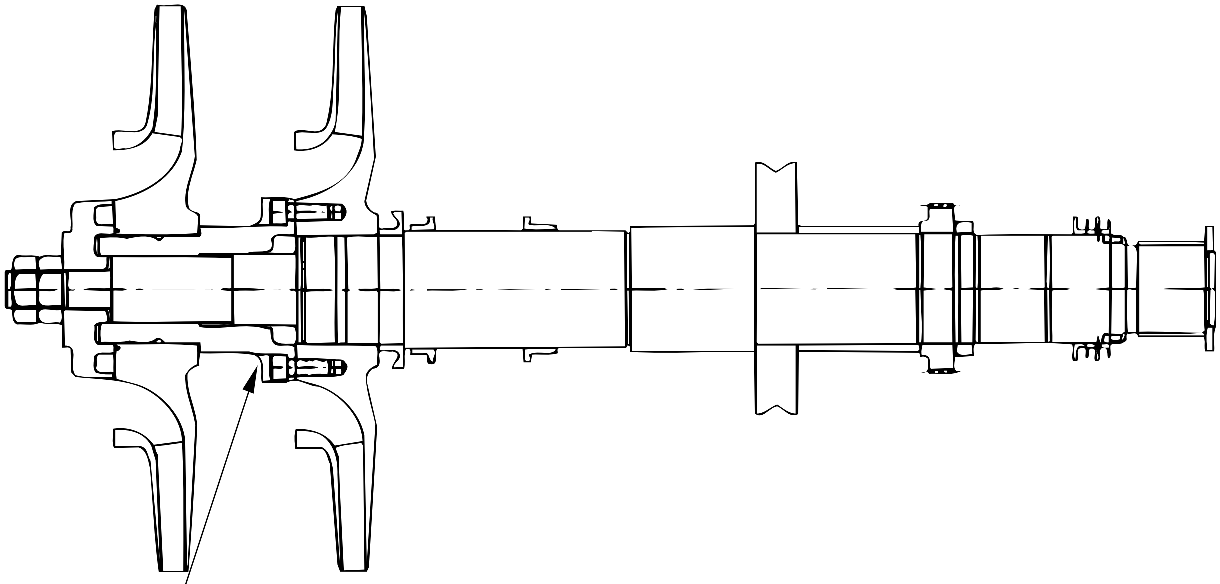
a. Armature windings in electric motors.
b. Shrunk-on gear meshes.
c. Wet impeller mass and inertia in pumps.

It is imperative that the rotor model properly account for these masses and any additional rotating masses that may be peculiar to a particular system.

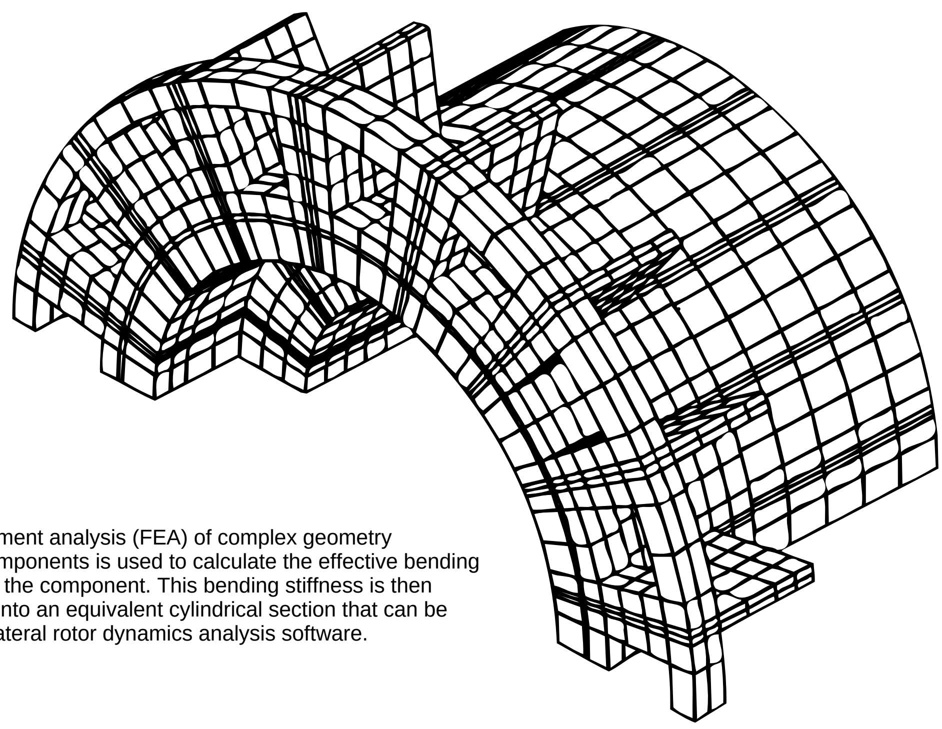
1.5.2.3 Addition of Stiffening Due to Shrink Fits and Irregular Sections

Most rotating assemblies have non-integral collars, sleeves, impellers, and so forth, that are shrunk onto the shaft during rotor assembly. As noted in the preceding, these shrunk-on components generally do not contribute to the lateral stiffness of the shaft. In some cases, however, if the amount and length of the shrink fit and the size of the shrunk-on component are sufficiently large, then the shrunk-on component must be modeled as contributing to the shaft stiffness. The vendor must determine the importance of shrink fits for particular cases. Often, this can be accomplished only by experience with units of similar type. A modal test of a vertically hung rotor will give some indication of the stiffening effect of shrunk-on components, but such measurements will likely exaggerate such effects because the fits will tend to be relieved through centrification at normal operating speeds.

Non-circular rotor cross-sections are common in the midspan areas of electric motors and generators. These electrical machines frequently possess integral or welded-on arms in the midspan area to support the rotor core. These



Cross-Section of Rotating Element with Complex Geometry Component



A finite element analysis (FEA) of complex geometry rotating components is used to calculate the effective bending stiffness of the component. This bending stiffness is then converted into an equivalent cylindrical section that can be input into lateral rotor dynamics analysis software.

3D Finite Element Model of Stub Shaft

Figure 1-15—3D Finite Element Model of a Complex Geometry Rotating Component structures add significant stiffening to the rotor midspan. This contribution to the lateral bending stiffness of the rotating assembly must be accounted for, as it is incorrect to model the stiffness of motor rotors using the base shaft only. Older steam turbines of built-up construction may also possess non-circular midspan rotor cross-sections.

1.5.2.4 Location of Bearings and Seals

It is well understood that bearings and seals can dramatically alter the vibration behavior of a rotating machine. It follows that these coefficients must be accurately placed in the rotor model for the numerical simulation to generate accurate results. Each fluid film support bearing or floating ring oil seal is typically represented using a set of eight linearized dynamic coefficients. The linearized models of the bearings and seals are assumed to act at the centerlines of the associated bearing and sealing lands.

1.5.2.5 Determination of Material Properties

The material properties required to generate the model are presented in Table 1-1.

Table 1-1—Typical Units for Material Properties

Quantity	Typical SI Units	Typical US Customary Units
Gravitational acceleration (g)	9.88m/s^2	386.4 in/s^2
Mass density (ρ)	kg/m^3	lbf·s^2/in^4
Weight density (ρg)	N/m^3	lbf/in^3
Young's modulus (E)	N/m^2	lbf/in^2
Shear/Rigidity modulus (G)	N/m^2	lbf/in^2

Results of the complete modeling process are displayed for an eight-stage 12-megawatt (16,000-horsepower) steam turbine rotor. A cross-sectional drawing of the unit is displayed in Figure 1-16. A larger version of this drawing was used to describe shaft geometry. The measured rotor weight was used to check the results of the modeling process. The resulting tabular description of the model is presented in Table 1-2. Note that the translational and rotational inertias shown in this table are formed by the sum of externally applied inertias (from turbine blades and disks) and shaft inertias calculated for each of the shaft sections. A cross section of the rotor model is displayed in Figure 1-17.

1.5.3 BEARING MODELS

By API requirement, most petroleum process turbomachinery must contain pressurized oil film bearings. Although this type of bearing is commonly called a *sliding bearing*, the rotor actually rides on a thin film of oil that separates the rotating shaft from the stationary bearing surface. Two common types of oil film bearings are presented in Figure 1-18. The two axial groove bearing is an example of a fixed bore sleeve bearing; other sleeve bearing designs may have different configurations for the oil grooves and have pocketed regions in the main bearing surfaces. The large variety of this type of bearing illustrates the different design requirements, design philosophies, and degree of sophistication of the unit's manufacturer. In the past twenty years, tilting pad bearings have found favor in the process industry. The tilting pad bearing has multiple pads which can rotate on pivot lines or points, allowing the bearing to adjust to the change in both steady state and transient journal position during operation.

Figure 1-19 displays an exaggerated schematic of an operating journal bearing. Note that the operating position of the journal is located below the bearing centerline. In most of the top half of the bearing, the oil film is cavitated because the thickness of the oil film is divergent (increases in the direction of shaft rotation) in this area. In most of the bottom half of the bearing, the film thickness converges (decreases in the direction of shaft rotation) so a pressurized oil film *wedge* forms to support the rotating journal. Note that for the journal to attain a static equilibrium position in the bearing, the oil film forces (integrated pressures) must balance in both the horizontal and vertical directions. For this reason, the rotating shaft does not displace solely in the vertical downward (-y) direction but also displaces sideways in the positive horizontal (+x) direction.

The displacement of the journal from the center of the bearing is a nonlinear function with the applied load. The linearized bearing stiffnesses (rate of change of force with position) of the oil film (k_{xx}, k_{xy}, k_{yx}, and k_{yy}) are, therefore, also nonlinear functions of the journal's equilibrium position. The linearized damping coefficients generated by the bearing are similarly nonlinear functions of the journal's equilibrium position. Thus, the linearized bearing dynamic coefficients (c_{xx}, c_{xy}, c_{yx}, and c_{yy}) are not solely dependent on the bearing geometry, but also on the applied bearing load. Ranges of stiffness and damping coefficients for various bearing designs are given in Table 1-3. The differences in the bearing dynamic characteristics with bearing type and applied load can make a great difference in the lateral rotor dynamic characteristics of a given machine.

In order to establish the effect that bearings have on the dynamic characteristics of a rotor system, the linearized bearing dynamic coefficients must be calculated. Several commercial and university computer codes are currently available that enable an engineer to calculate the bearing coefficients for a particular bearing. These codes are usually based on either the finite element or finite difference methods for solving Reynolds' equation, the governing two-dimension differential equation for thin hydrodynamic films. Such codes are able to account for a variety of effects including the effect of heat generation due to fluid shearing in the film, fluid turbulence in the film, variation in oil supply temperatures, and so on. The net result of the computer analysis of the bearing is the set of eight linearized hydrodynamic bearing coefficients, as presented in Figure 1-20 and de-

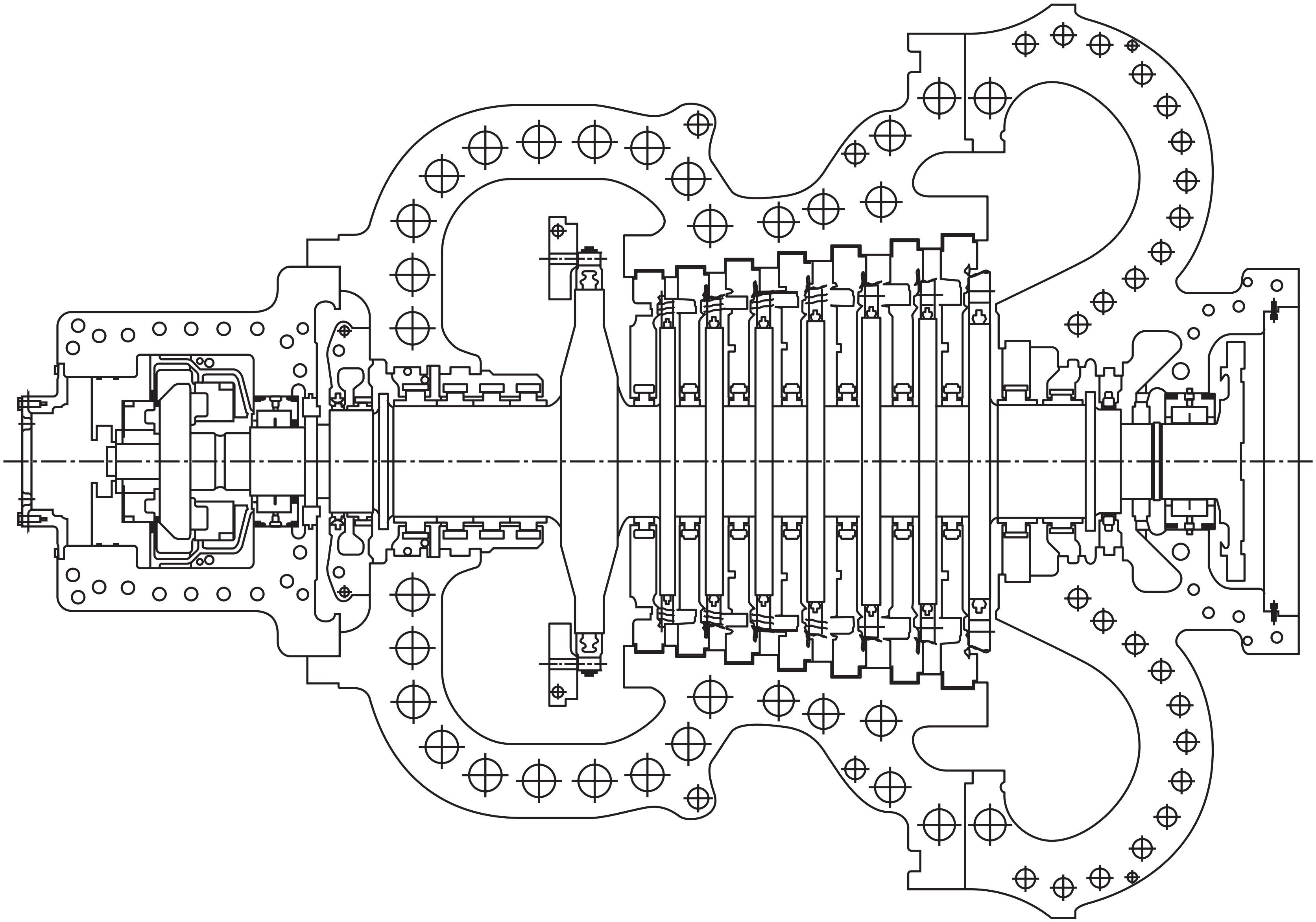


Figure 1-16—Cross Sectional View of an Eight-Stage 12 MW (16,000 HP) Steam Turbine

Table 1-2—Tabular Description of the Computer Model Generated for the 12 MW (16,000 HP) Eight-Stage Stream Turbine Rotor (US Customary Units)

Station No.	Axial Displacement (in.)	Weight (lb.)	Length (in.)	Shaft Diameter Outside	Shaft Diameter Inside	I (in.^{4})	I_P Mom. (lb-in.^{2})	I_T Mom. (lb-in.^{2})	(lb-in.^{2}) E*10^{-6}
1	.00	1.762	1.500	3.250	.000	5.48	2.321	1.491	28.700
2	1.50	3.523	1.500	3.250	.000	5.48	4.652	2.991	28.700
3	3.00	3.523	1.500	3.250	.000	5.48	4.652	2.991	28.700
4	4.50	23.786	1.300	12.345	.000	1140.08	421.909	214.386	28.700
5	5.80	47.029	1.200	13.690	.000	1724.18	1005.325	508.760	28.700
6	7.00	29.793	1.723	5.000	.000	30.68	600.712	304.538	28.700
7	8.72	9.579	1.723	5.000	.000	30.68	29.941	17.336	28.700
8	10.45	9.579	1.723	5.000	.000	30.68	29.941	17.336	28.700
9	12.17	13.195	2.100	6.000	.000	63.62	52.789	30.671	28.700
10	14.27	17.970	2.390	6.000	.000	63.62	80.869	48.077	28.700
11	16.66	17.007	1.190	7.500	.000	155.32	95.364	53.119	28.700
12	17.85	12.185	1.010	6.500	.000	87.62	77.377	39.974	28.700
13	18.86	43.105	4.260	9.000	.000	322.06	413.466	265.154	28.700
14	23.12	117.719	5.680	10.000	.000	490.87	906.536	563.578	28.700
15	28.80	109.175	4.910	10.000	.000	490.87	1364.690	901.689	28.700
16	33.71	109.172	4.910	10.000	.000	490.87	1364.652	901.648	28.700
17	38.62	515.401	2.500	15.000	.000	2485.05	1402.152	811.401	28.700
18	41.12	515.404	2.500	15.000	.000	2485.05	2629.723	1316.594	28.700
19	43.62	59.702	3.570	10.000	.000	490.87	1215.937	650.785	28.700
20	47.19	138.463	1.250	11.250	.000	786.28	9424.444	4816.282	28.700
21	48.44	127.568	2.590	10.000	.000	490.87	9288.255	4722.129	28.700
22	51.03	157.523	1.570	11.570	.000	879.64	12322.275	6252.180	28.700
23	52.60	158.524	2.680	10.000	.000	490.87	12334.790	6260.173	28.700
24	55.28	162.368	1.560	11.560	.000	876.60	13086.088	6636.282	28.700
25	56.84	165.258	2.940	10.000	.000	490.87	13122.231	6660.061	28.700
26	59.78	170.604	1.570	11.570	.000	879.64	14030.082	7115.411	28.700
27	61.35	171.937	3.060	10.000	.000	490.87	14046.758	7126.755	28.700
28	64.41	174.432	1.570	11.570	.000	879.64	14517.968	7362.875	28.700
29	65.98	177.100	3.300	10.000	.000	490.87	14551.340	7386.304	28.700
30	69.28	177.100	1.570	11.570	.000	879.64	14551.330	7386.294	28.700
31	70.85	170.763	2.730	10.000	.000	490.87	14472.132	7332.254	28.700
32	73.58	197.509	1.880	11.880	.000	977.77	17066.588	8649.229	28.700
33	75.46	211.795	4.015	10.000	.000	490.87	17245.151	8779.615	28.700
34	79.47	146.058	4.785	10.000	.000	490.87	1115.942	677.903	28.700
35	84.26	32.353	2.240	9.000	.000	322.06	423.397	220.752	28.700
36	86.50	27.875	1.640	6.500	.000	87.62	244.917	132.617	28.700
37	88.14	15.406	1.640	6.500	.000	87.62	81.369	44.136	28.700
38	89.78	19.540	2.520	6.500	.000	87.62	103.196	59.591	28.700
39	92.30	24.801	2.760	6.500	.000	87.62	130.976	79.978	28.700
40	95.06	111.748	2.000	21.620	.000	10724.89	314.579	165.750	28.700
41	97.06	77.957	.000	21.620	.000	.00	4554.855	2292.055	28.700
		4475.292	97.059						

Bearing Reactions: 2190.92 lb at Station 10.
2284.38 lb at Station 39.

Table 1-3—Typical Stiffness and Damping Properties of Common Bearings (Comparison Only)

Bearing Type	Typical Stiffness (Comparative) N/mm	lbf/in.	Typical Damping (Comparative) N·s/mm	lbf·s/in.
Plain bushing bearings with and without oil grooves	87,565	500,000	350.3	2000
Multi-lobe insert bearings	122,591	700,000	262.7	1500
Pressure dam/multi-dam bearings	175,130	1,000,000	525.4	3000
Single pocket/multi-pocket bearings	175,130	1,000,000	525.4	3000
Hydrostatic bearings	210,156	1,200,000	560.4	3200
Tilting pad bearings (load on pad and load between pads)	131,348	750,000	131.4	750
Magnetic bearings	Variable	Variable	Variable	Variable
Anti-friction bearings	875,650	5,000,000	13.1	75

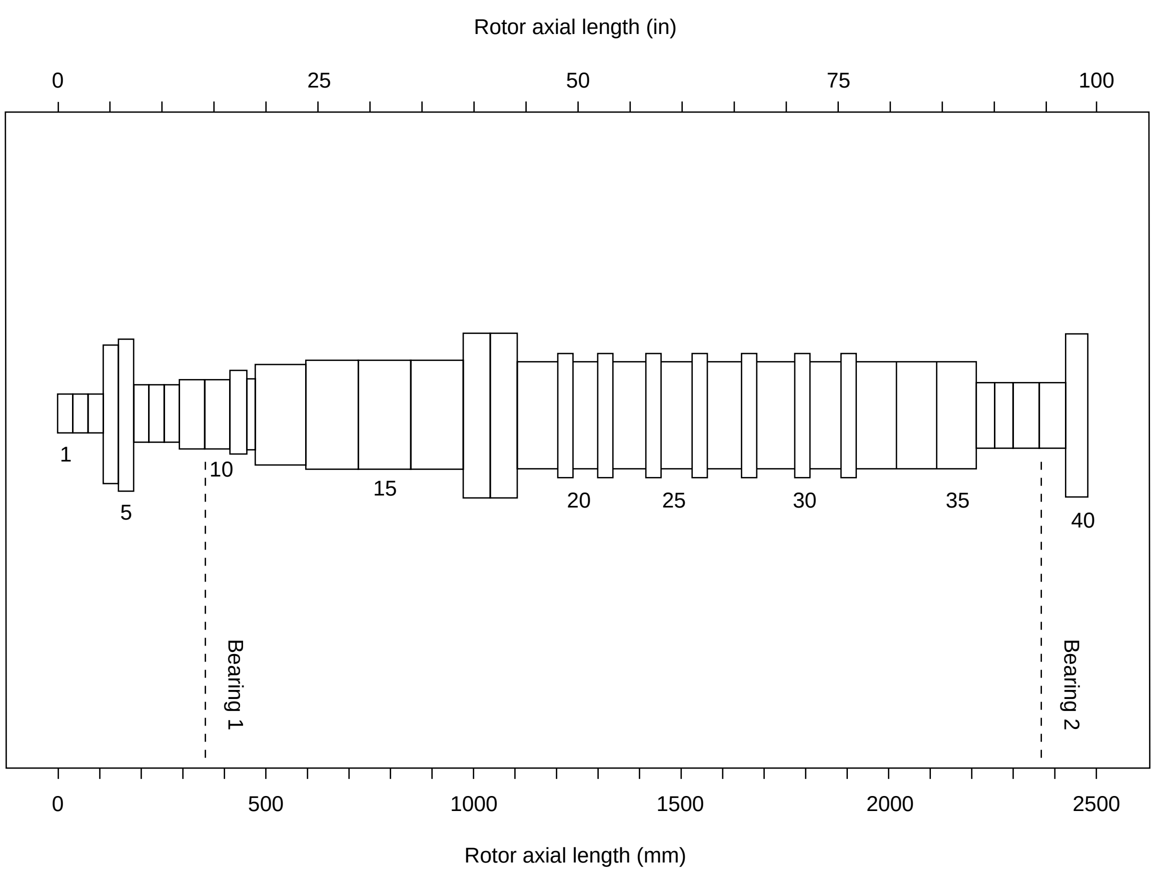
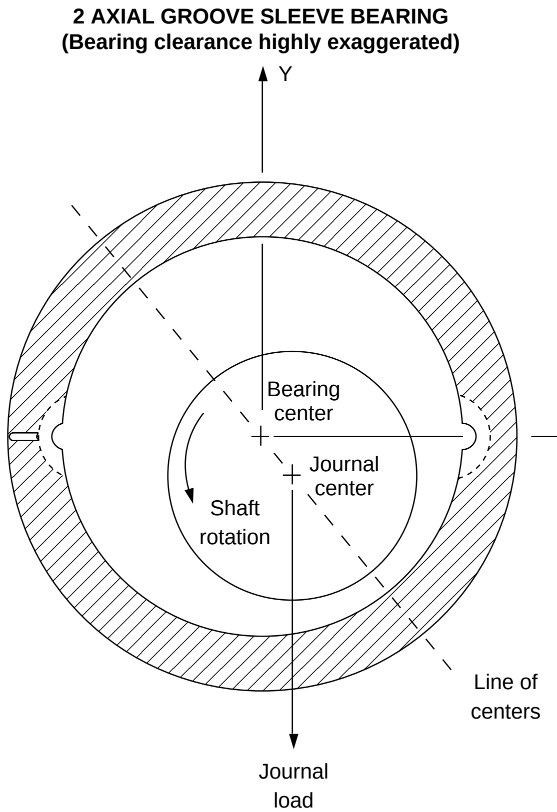
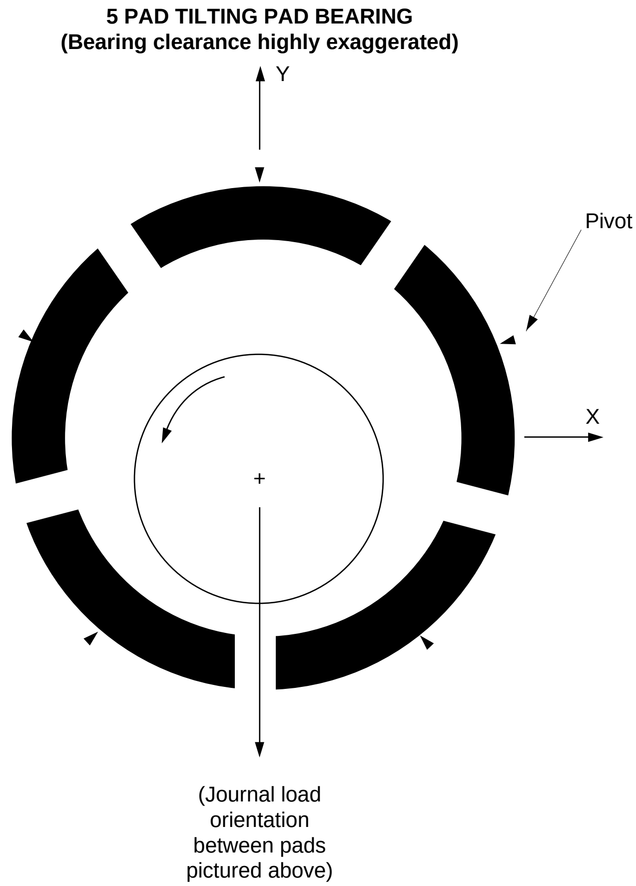


Figure 1-17—Rotor Model Cross Section of an Eight-Stage 12 MW (16,000 HP) Steam Turbine





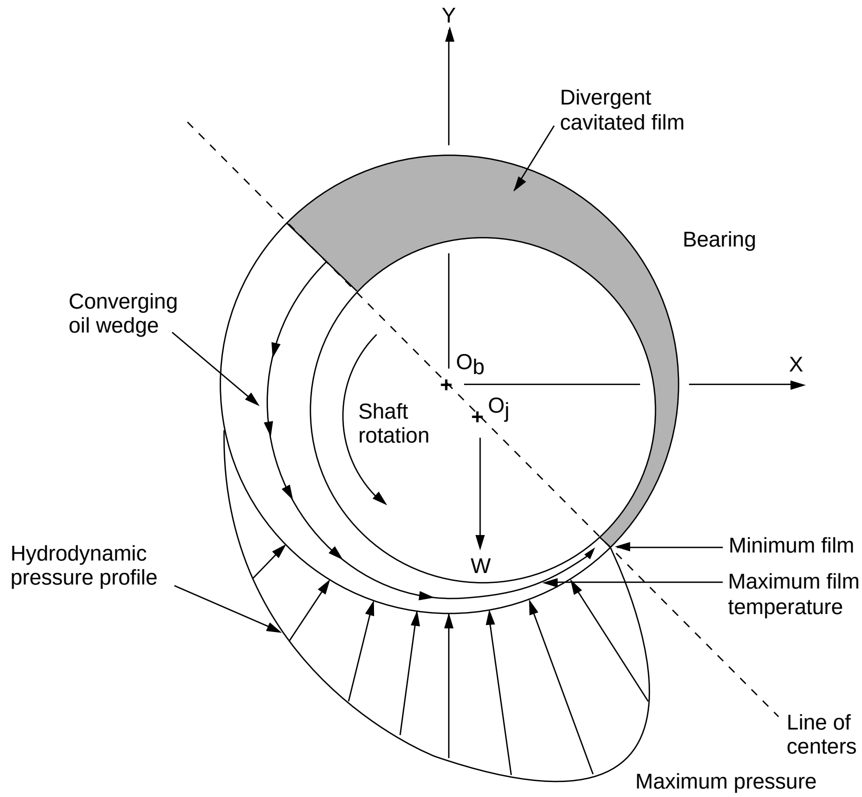
Typical Design Ranges for Bearing Geometry
2 Axial Groove Sleeve Bearing

Diametral Bearing Clearance	Approximately 0.0015 mm per mm of Journal Diameter (inches per inch)
L/D ratio	0.4 < L/D < 1.0

Typical Design Ranges for Bearing Geometry
5 Pad Tilting Pad Bearing

Diametral Bearing Clearance	Approximately 0.0015 mm per mm of Journal Diameter (inches per inch)
Pad preload	0.2 < m < 0.6
L/D ratio	0.4 < L/D < 1.0
Load orientation	Either Load-on-Pad or Load-Between-Pads

Figure 1-18—Examples of Two Common Bearing Designs



Notes:
1. O_b = Bearing center
2. O_j = Journal center

Figure 1-19—Hydrodynamic Bearing Operation (With Cavitation)

scribed in Figure 1-21. This modeling method is valid up to high bearing L/D (length-to-diameter ratio) ratios (L/D less than 1.0). Higher L/D bearings are commonly found in older steam turbines and centrifugal compressors and may have to be modeled using a set of sixteen linearized dynamic coefficients (translational coefficients described in the preceding, plus rotational dynamic coefficients).

The information required to perform a bearing analysis is listed in the following.

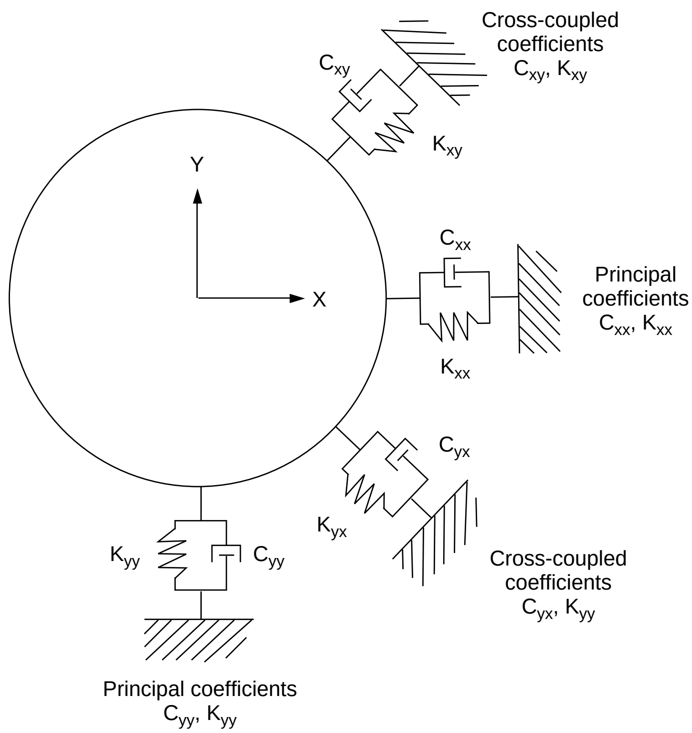
1.5.3.1 Geometric Dimensions

The geometric data found in Table 1-4 describe the bore or film profile of most fixed geometry or sleeve-type fluid film bearings. Most of this information is usually included on bearing drawings that are provided by the bearing manufacturer. Alternatively, the authors have enjoyed great success in reverse engineering bearing geometries by carefully measuring bore profiles.

Table 1-4—Input Data Required With Typical Units for Fixed Geometry Journal Bearing Analysis

Parameter	Typical SI Units	Typical US Customary Units
Bearing axial length	millimeters	inches
Journal diameter	millimeters	inches
Bearing clearance (C_D) (clearance bore minus the journal diameter)	micrometers	mils
Lobe offset	non-dimensional	non-dimensional
Lobe preload	non-dimensional	non-dimensional
Location and geometry of oil supply grooves	degrees, mm	degrees, inches
Location and geometry of dams, pockets, or tapered surfaces	degrees, µm	degrees, mils

Note: µ = micrometers; mm = millimeters



Result of a non-linear hydrodynamic bearing analysis:
eight linear stiffness and damping coefficients

Figure 1-20—Linear Bearing Model Used in Most Rotor Dynamics Analysis

The following table displays geometric parameters defining tilting pad bearings:

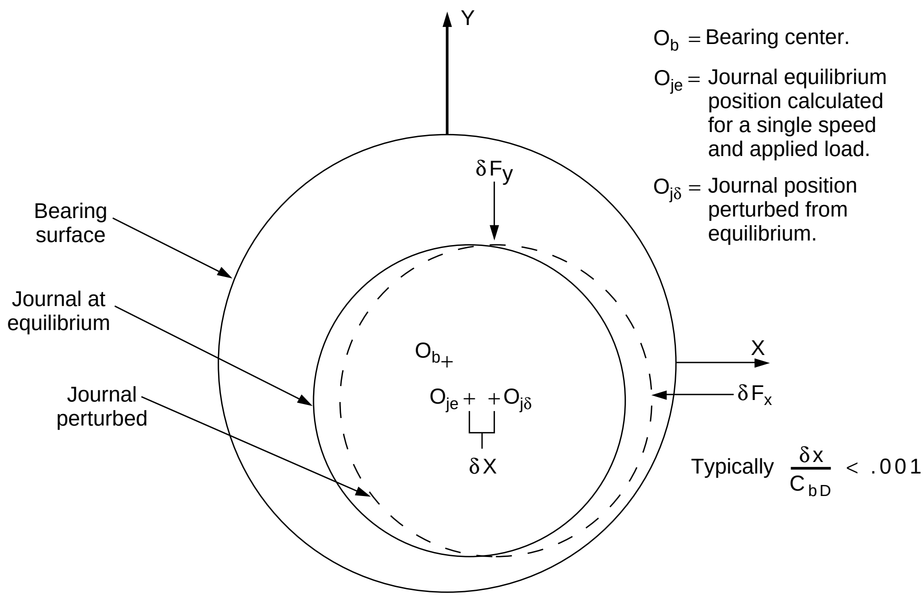
Table 1-5—Geometric Input Data Required With Typical Units for Titling Pad Journal Bearing Analysis

Quantity	Typical SI Units	Typical US Customary Units
Bearing axial length	millimeters	inches
Journal diameter	millimeters	inches
Load orientation (load on pad or load between pads)	non-dimensional	non-dimensional
Bearing clearance (C_{bD}) (bearing clearance bore minus journal diameter)	micrometers	mils
Pad clearance (C_{pD}) (ground pad bore minus journal diameter)	micrometers	mils
Pivot offset (0.50 denotes a centrally-pivoted bearing)	non-dimensional	non-dimensional
Pad arc length	degrees	degrees

Given the bearing and pad clearances, the preload of the pads may be calculated using the following equation (1-7):

$$\text{Preload} = 1 - \frac{C_{b(DIA)}}{C_{p(DIA)}} \tag{1-7}$$

The value of preload can significantly influence bearing coefficients, as illustrated in Figure 1-22. Note that manufacturing tolerances associated with the bearing and pad clearances may significantly change the preloads in manufactured bearings of the same design. This is especially true in small diameter (journal OD less than 50 millimeters or 2.0 inches) tilting pad bearings where manufacturing tolerances are nearly the same as in larger diameter bearings. For this reason, the pad preload in small diameter bearings may vary from 0.0 to 0.5. As a general rule, the bearing and pad clearances (including manufacturing tolerances) should always be selected to avoid manufacturing a bearing with negative preload. Negative preloads in tilting pad bearings should be avoided simply because the dynamic characteristics of such



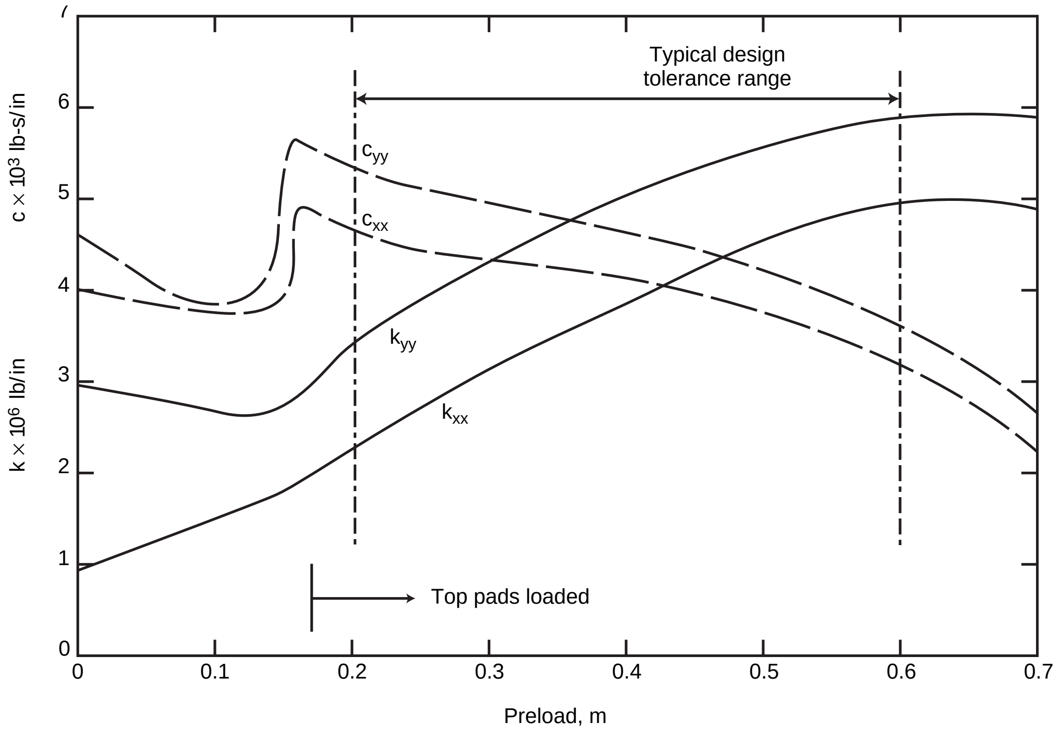
Note: The figure above displays how two of the four bearing stiffness coefficients K_{xx} and K_{yx} are calculated as shown below:

1. Determine the journal equilibrium position at the specified speed and load.
2. Perturb the journal a small horizontal distance (δx) from the journal equilibrium position and calculate the change in force in the horizontal (x) and vertical (y) directions.
3. K_{xx} and K_{yx} are defined as follows:

$K_{xx} = -\delta F_x / \delta_x \qquad K_{yx} = -\delta F_y / \delta x$

4. The other two bearing stiffness coefficients are similarly calculated in pairs by perturbing the journal displacement in the vertical (y) direction and calculating the resulting change in forces on the journal.
5. The four bearing damping coefficients are calculated in similar fashion to the stiffness coefficients except that the velocities are perturbed in the x and y directions instead of the displacements.

Figure 1-21—Calculating Linearized Bearing Stiffness and Damping Coefficients



Note: $m = 1 - \frac{c_b}{c_p}$

Figure 1-22—Effect of Preload on Tilting Pad Bearing Coefficients

a bearing are generally adverse to the lateral rotor dynamics characteristics of a turbomachine for the following reasons:

a. The damping provided by the bearing decreases sharply when the pad preloads decrease to near zero.
b. Tilting pad bearings with negative preloads can actually cause rotor instability. For example, when the radius of curvature of the pad is less than the radius of the journal, a converging oil film capable of supporting the rotor cannot be generated.

1.5.3.2 Bearing Loads

The static journal load applied to each bearing is a function of the mass distribution of the rotating element. In addition, other static load mechanisms exist that contribute additional bearing loadings and must be included in the bearing analysis. Some common sources of additional loading include the following:

a. Static gear loads.
b. Static loads that result from partial arc admission in steam turbines.
c. Lateral-torsional load coupling in misaligned couplings.

1.5.3.3 Oil Properties

Although most petroleum companies manufacture lubricants suitable for use in turbomachinery, most lubricants possess similar fluid dynamic and thermodynamic properties. High speed turbomachinery (greater than 2000 revolutions per minute) is generally lubricated using light turbine oil (32 centistokes at 37.7°C or 150 Standard Saybolt Universal (SSU) at 100°F), while lower speed equipment (for example, four-pole electric motors and generators) may be lubricated using a higher viscosity oil. Characteristics of the lubricant relevant to hydrodynamic bearing analysis include those listed in Table 1-6.

Table 1-6—Lubricant Data Required With Typical Units for Journal Bearing Analysis

Quantity	Typical SI Units	Typical US Customary Units
Viscosity	centipoise	Reyns
Density	kg/m^3	$lbf•s^2/in^4$
Specific heat	kJ/(kg•˚C)	BTU•in/(lbf•s^2•˚F)
Thermal conductivity	W/(m•˚C)	BTU/(in•s•˚F)

For isothermal bearing analysis, the thermal changes through the film are ignored and a bulk viscosity is adopted over the complete film. The standard bulk viscosity used for such analysis is 9.65 centipoise (1.4 x 10-6 Reyns).

1.5.3.4 Results of Bearing Analysis

In addition to determining the stiffness and damping coefficients, some bearing programs are capable of calculating a number of additional parameters. Some of these parameters are used in the design process to evaluate a particular design. These useful parameters include those listed in Table 1-7.

Table 1-7—Results of a Journal Bearing Analysis With Typical Units

Quantity	Typical SI Units	Typical US Customary Units
Average film temperature	°C	°F
Maximum film temperature	°C	°F
Reynolds number	non-dimensional	non-dimensional
Sommerfeld number	non-dimensional	non-dimensional
Minimum film thickness	micrometer	mils
Operating eccentricity	micrometer	mils
Equilibrium attitude angle	degrees	degrees
Power loss	kilowatt	HP
Oil flowrates	liters/min	gal/min
Average/bulk viscosity	centipoise	Reyns

1.5.4 OIL SEAL MODELING

In centrifugal compressors, floating ring oil seals can act like radial bearings and affect rotor response characteristics. Oil seals are designed to keep process fluids from discharging into the atmosphere by placing a barrier between the process gas and the atmosphere. A typical single breakdown liquid-film shaft seal with cylindrical bushings is shown in Figure 1-23. A diagram of the outer sealing ring is presented in Figure 1-24, showing key dimensions and the general oil pressure distribution. These rings generally *lock-up* in a set radial position when the unit operates because the unbalanced pressures produce a net axial load that creates a radial friction force that opposes motion of the otherwise-floating bushing or ring. After the floating rings *lock-up* under the radial friction forces, they effectively operate like plain sleeve bearings and generally alter the dynamic behavior of the compressor in as significant a way as the fluid film bearings. For this reason, API mandates that the effect of the oil seals be considered as an integral part of the damped response analysis.

The pressure field of the oil film in the seals is described using the same differential equation employed to describe

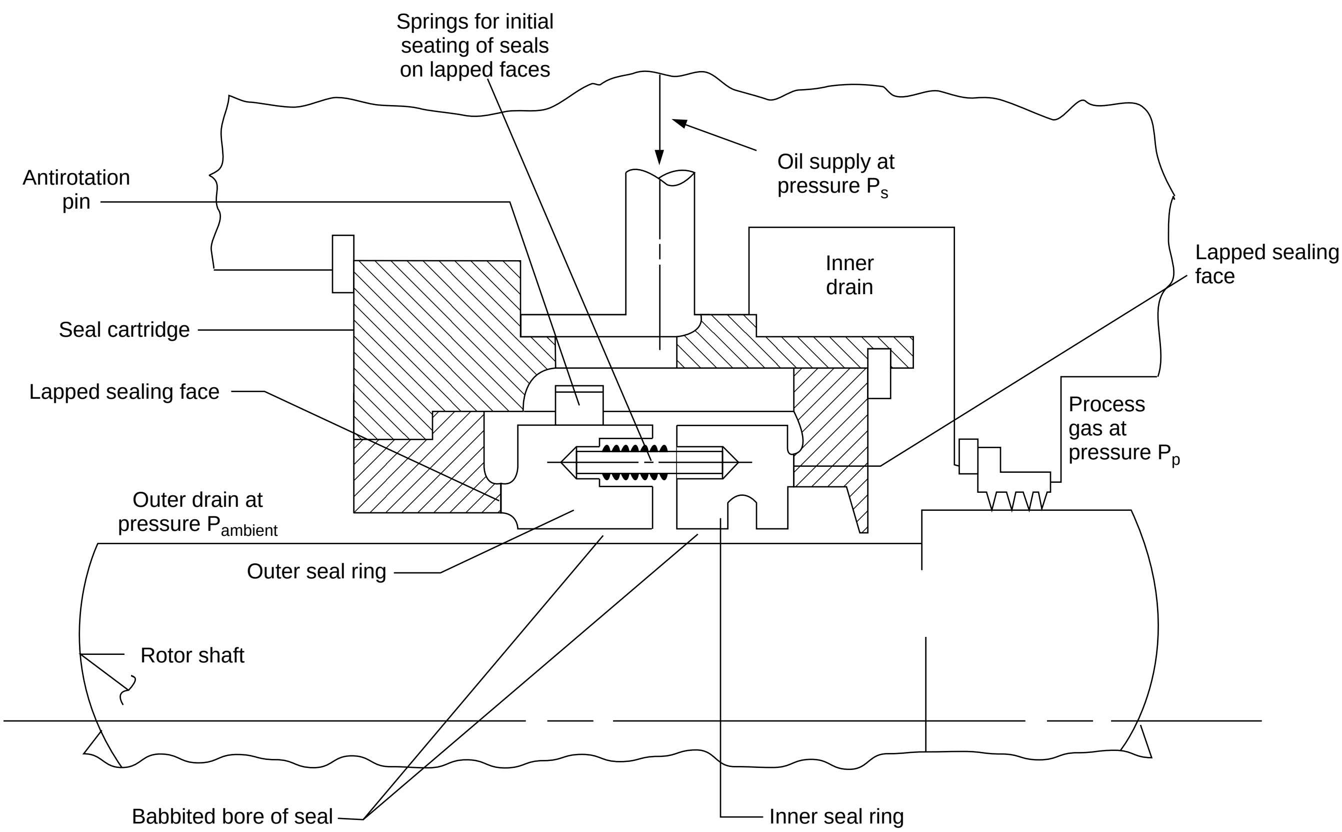
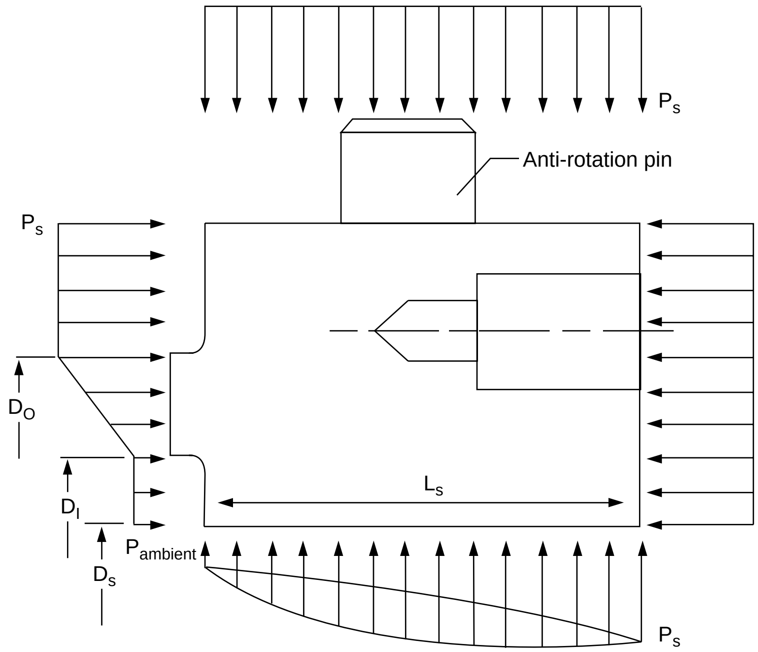


Figure 1-23—Oil Bushing Breakdown Seal



Notes:

P_s = seal oil supply pressure.
$P_{ambient}$ = ambient pressure.
D = diameter of sealing land.
D_I = inner diameter of lapped sealing face.
D_O = outer diameter of lapped sealing face.
L_s = length of sealing land.

Figure 1-24—Pressures Experienced by an Outer (Low Pressure) Floating Oil Seal Ring During Operation

the oil film pressures in hydrodynamic journal bearings. The principal difference between the numerical methods used to analyze bearings and seals lies solely in the equilibrium calculations. In hydrodynamic bearing analysis, the equilibrium position of the journal is determined by balancing the static load applied by the journal to the bearing, with the force generated by the oil film. In liquid film seal analysis, the equilibrium position of a floating ring is determined by balancing the radial forces generated by the oil film, with the radial friction load stemming from the unbalanced axial pressure. Once the equilibrium positions of the journal or the floating ring are evaluated, linearized dynamic coefficients describing the influence of the seal on the rotating journal may be calculated by perturbing position and velocity of the rotating journal. Note that turbulence and fluid inertia effects in the oil film are frequently neglected. Such terms are generally not negligible in water film seals.

As previously noted, the floating ring oil seals may exert great influence on the lateral dynamic characteristics of a centrifugal compressor. For example, the damped response of the hydrogen recycle compressor rotor displayed in Figure 1-25 to a general unbalance distribution has been calculated for various start-up sealing pressures. The midspan response amplitudes for this compressor through the first critical are displayed in Figure 1-26 for three start-up sealing pressures. Note that as the sealing pressure increases, the influence of the oil seals increases because the damping generated by the seals increases. In general, the damping provided by oil seals tends to reduce the amplification associated with the fundamental critical and to raise the frequency of the critical speed.

Although the effect of oil seals on damped rotor response characteristics is extremely positive, oil seals may prove quite detrimental to the unit's rotor stability at operating speed and cause large amplitude subsynchronous vibrations. Much design effort has focused on minimizing the destabilizing effect of the seals. Oil seals are generally pressure balanced to minimize axial forces; the sealing lands are

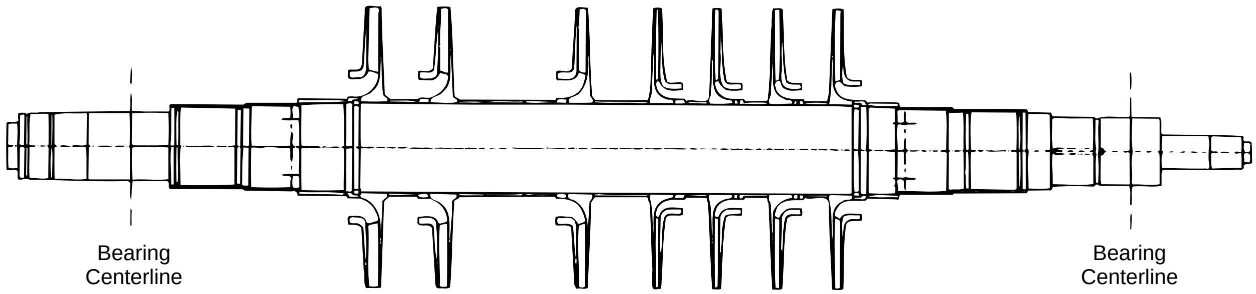


Figure 1-25—Seven Stage High Pressure Natural Gas Centrifugal Compressor

frequently grooved in order to diminish hydrodynamic load capacity; and various centering mechanisms are used to reduce the eccentricity of the ring, relative to the shaft during operation. Extra care must also be taken when different seals and pressures are used during mechanical acceptance testing compared to the field operation. In some cases, the difference between test seals installed during the mechanical acceptance test and the seals installed during field operation has proved great enough to drive the system unstable and prevent the unit's safe operation. If job and test seals are different, then both sets of seals should be analyzed.

1.5.4.1 Seal Type

In order to analyze seals properly, the type of oil seal must be identified and the particular design characteristics understood. Common types of casing end seals are displayed in API Standard 617, Fifth Edition, and are duplicated in Figures 1-27 through 1-32. The seals listed by API include the following:

a. Labyrinth shaft seal.
b. Mechanical (contact) shaft seal.
c. Restrictive-ring shaft seal.

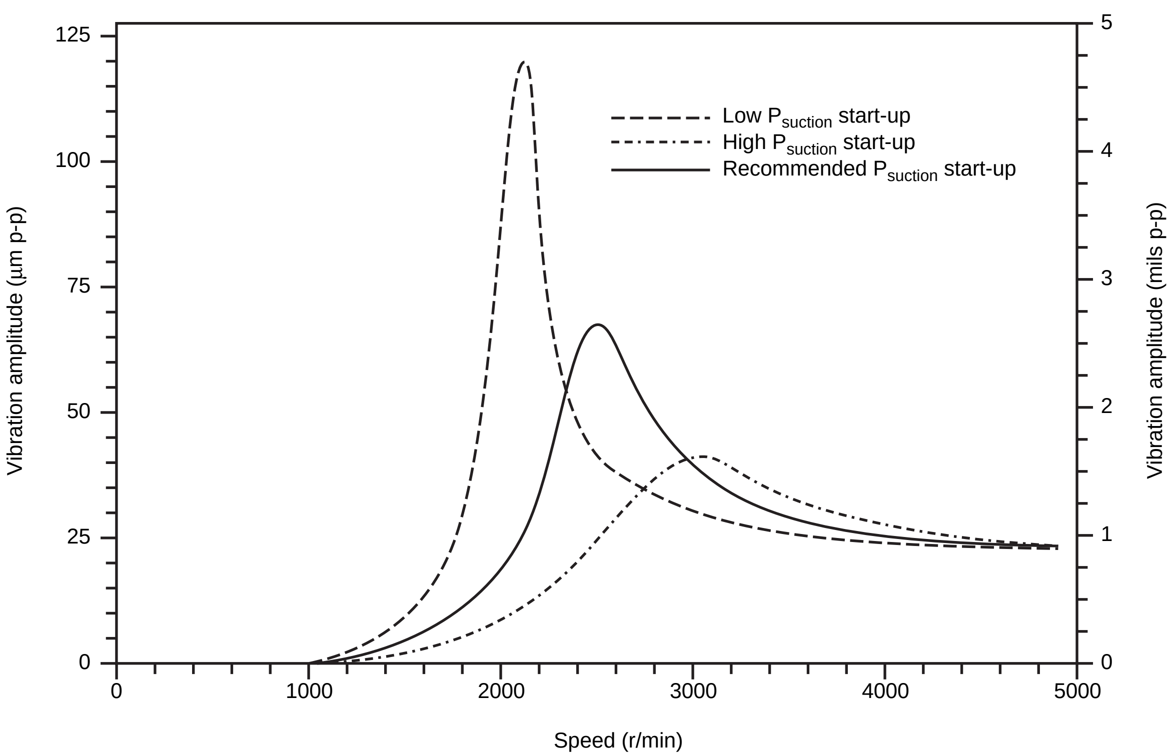


Figure 1-26—Midspan Rotor Unbalance Response of a High Pressure Centrifugal Compressor for Different Suction Pressures on Startup

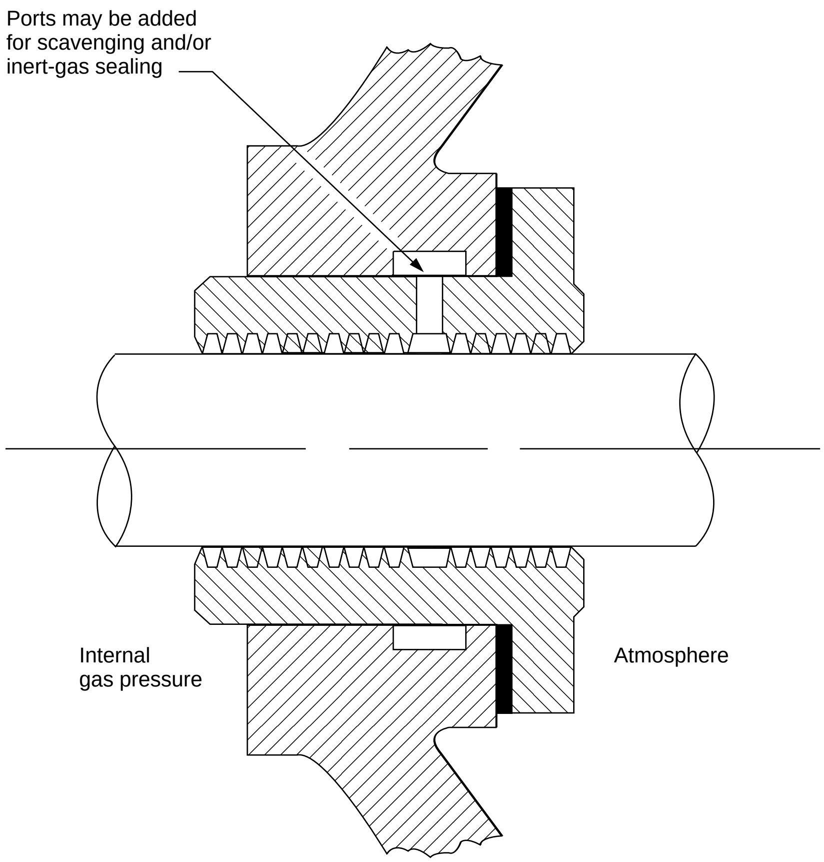


Figure 1-27—Labyrinth Shaft Seal

d. Liquid-film shaft seal with cylindrical bushing.
e. Liquid-film shaft seal with pumping bushing.
f. Self-acting gas seal.

Of the six types of seals listed above, only the seals with a fluid (typically oil) film between the rotating assembly and the non-rotative free-floating components (mechanical contact shaft seal, liquid-film shaft seal with cylindrical bushing, and liquid-film shaft seal with pumping bushing) affect the rotor dynamic characteristics of centrifugal compressors in the manner described above. Although restrictive-ring shaft seals may greatly affect lateral rotor vibrations, these seals are usually non-problematic and non-influential if properly broken in. Gas seals that are retrofit into a centrifugal compressor will often significantly affect the rotor dynamic characteristics of the unit by removing oil-film seal effects and by adding mass to the rotating assembly.

1.5.4.2 Geometric Dimensions

The geometric data listed in Table 1-8 is required to analyze floating oil seal rings and is usually included on a drawing or sketch of the seals.

Table 1-8—Geometric Input Data Required With Typical Units for Hydrodynamic Seal Analysis

Quantity	Typical SI Units	Typical US Customary Units
Number of seal rings	non-dimensional	non-dimensional
Axial seal length	millimeters	inches
Location and geometry of circumferential grooves in the sealing lands	millimeters	inches
Journal diameter	millimeters	inches
Inner and outer diameters of the sealing face or lip	millimeters	inches
Radial seal clearance	micrometers	mils

1.5.4.3 Oil Characteristics

As in the analysis of hydrodynamic fluid film bearings, the mechanical and thermodynamic properties of the lubricant must be used in the oil seal analysis. Characteristics of the lubricant relevant to hydrodynamic oil seal analysis are displayed in Table 1-9.

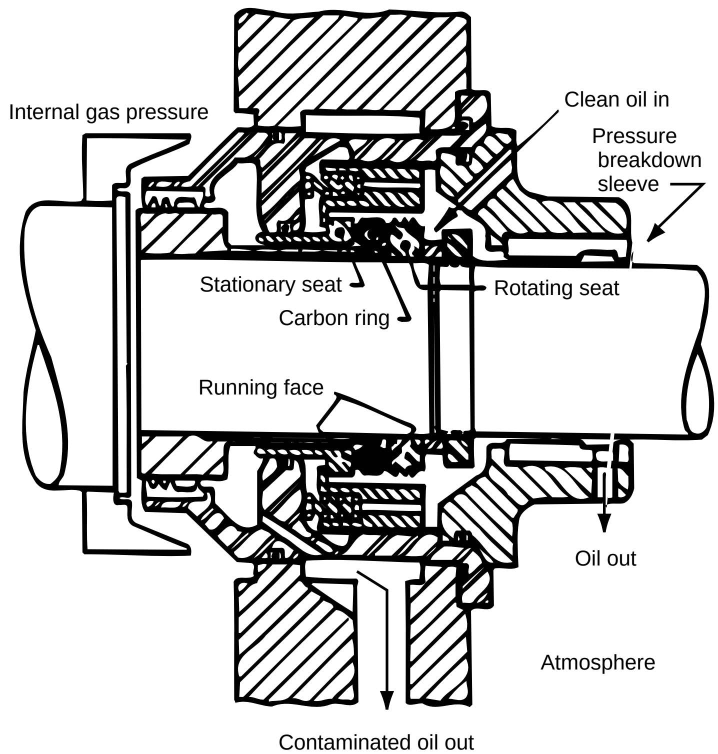


Figure 1-28—Mechanical (Contact) Shaft Seal

Table 1-9—Lubricant Data Required With Typical Units for Hydrodynamic Seal Analysis

Quantity	Typical SI Units	Typical US Customary Units
Viscosity	centipoise	Reyns
Density	kg/m^3	$lbf•s^2/in^4$
Specific heat	kJ/(kg•°C)	BTU•in/(lbf•s^2•°F)
Thermal conductivity	W/(m•°C)	BTU/(in•s•°F)

For isothermal analysis, the thermodynamic aspects of the lubricant film are ignored, and a bulk viscosity is adopted over the complete film. The standard bulk viscosity used for such analysis is 9.65 centipoise (1.4 x 10^{-6} Reyns).

1.5.4.4 Sealing Pressure

The influence of sealing pressure on the calculated rotor dynamic behavior of a centrifugal compressor was briefly discussed in 1.5.4. The results plotted in Figure 1-26 indicate that the seals become more influential as the sealing pressure increases. The sealing pressure may be defined as the pressure of the lubricant in the cavity or plenum between the high- and low-pressure floating seal rings. Typically, the sealing pressure is set by the manufacturer of the equipment to be between 69 and 103 kilopascals (10 and 15 pound-force/ $inches^2$) above the inlet pressure of the unit. This small pressure differential between the sealing pressure, results in a small amount of leakage through the inner/high pressure ring. Unless a chemically inert buffer gas is injected between the inner seal ring and the main labyrinth seal, the oil leaking past the high pressure sealing ring may chemically react with the process gas and form *sour* leakage that must be drained and degassed and either recycled or discarded. Most seal oil flows through the outer/low pressure seal where the major pressure drop occurs. As the oil leaking past the low pressure seal is not chemically contaminated by the process gas, this oil is returned to the seal oil supply system.

1.6 API Specifications and Discussion

1.6.1 GENERAL

Now that basic lateral rotor dynamics terminology, concepts, and analysis methods have been introduced, detailed

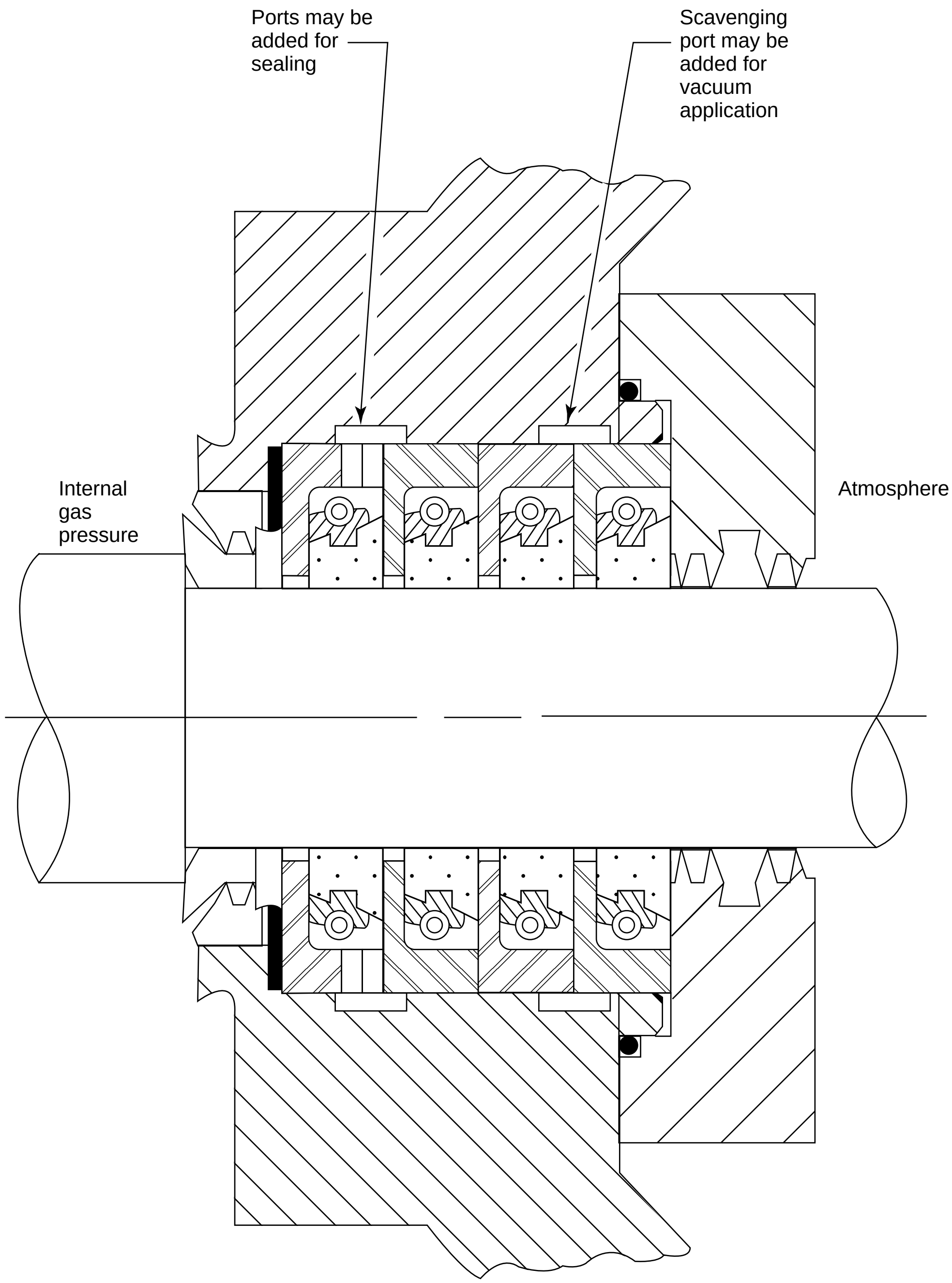


Figure 1-29—Restrictive-Ring Shaft Seal

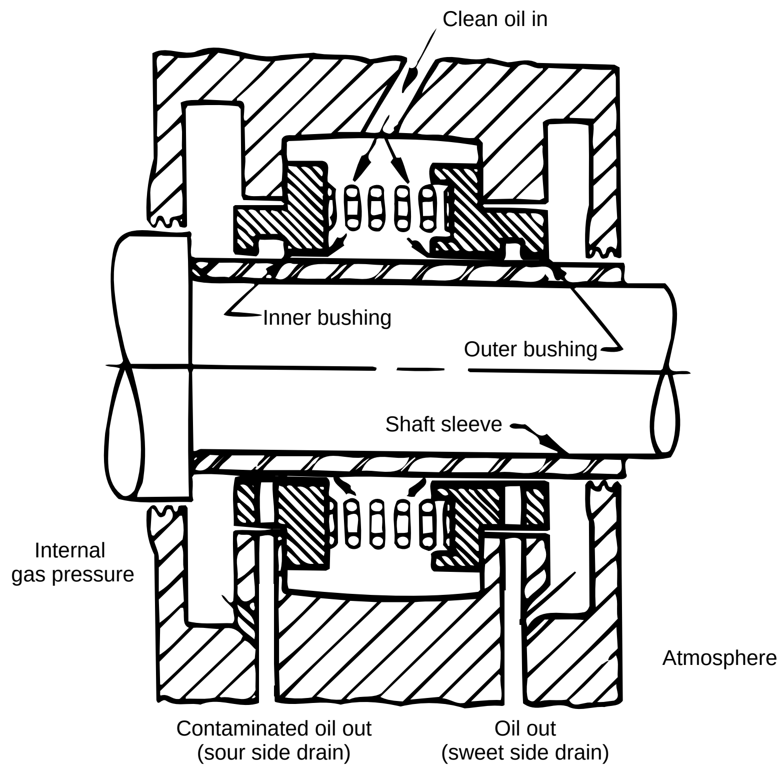


Figure 1-30—Liquid-Film Shaft Seal With Cylindrical Bushing

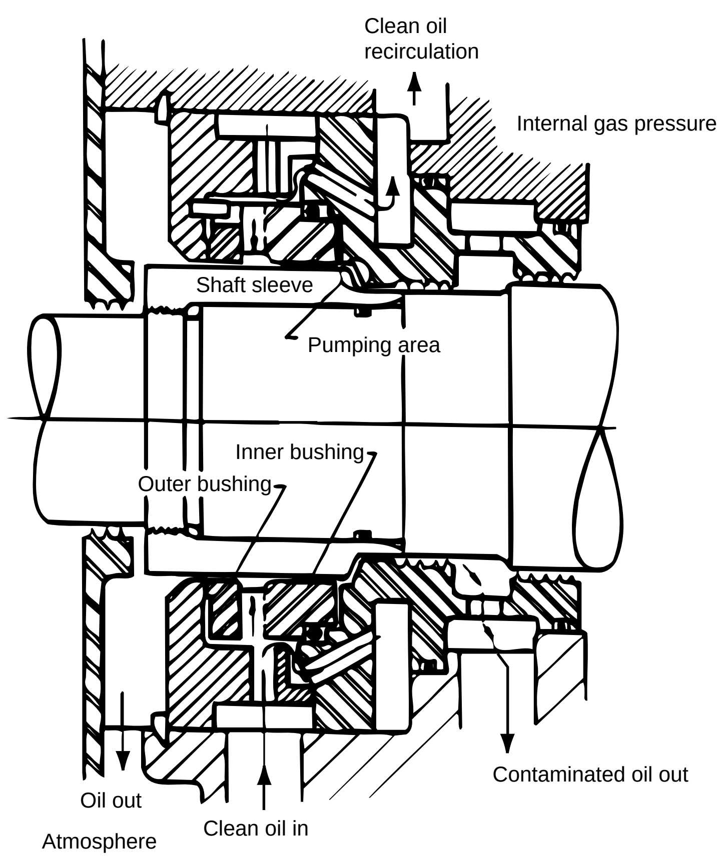


Figure 1-31—Liquid-Film Shaft Seal with Pumping Bushing

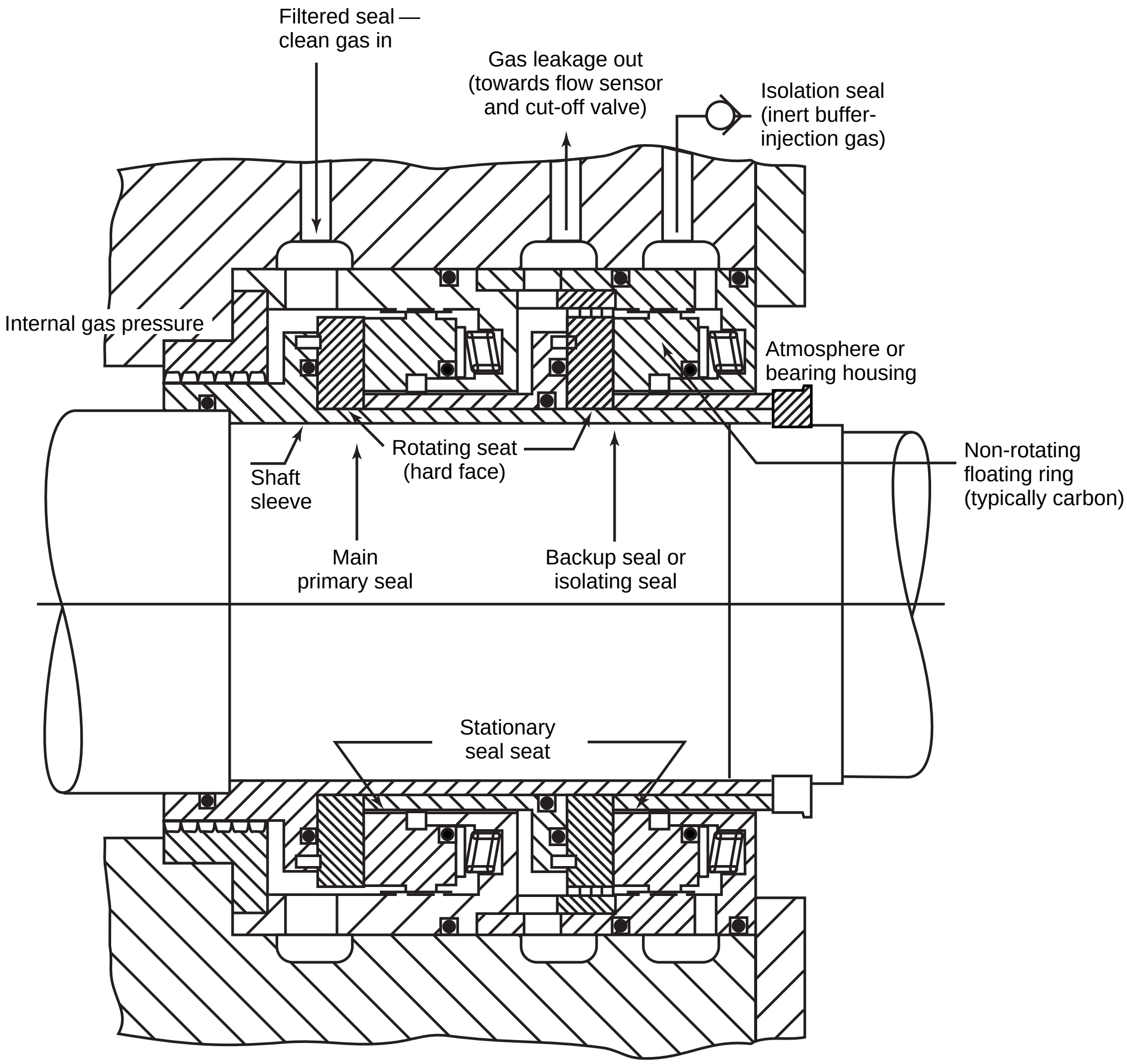


Figure 1-32—Self-Acting Gas Seal

discussion of individual API *Standard Paragraphs* relating to lateral rotor dynamics can proceed. The following format is employed for this discussion: each of the paragraphs comprising the latest revision of the API *Standard Paragraphs* are individually reproduced, in sequence, followed by commentary designed to illustrate or clarify the material contained in the paragraph. For clarity, the standard paragraphs have been reproduced in bold type and the paragraph numbers are preceded by *SP*, while the commentary immediately following the paragraph is printed in normal type.

As previously noted, the latest revision of the API rotor dynamics acceptance program described by the API *Standard Paragraphs* (see Appendix 1A) provide the petrochemical industry with a program that integrates computer analysis with vibration measurements recorded during the unit's mechanical run test. This program can be divided into three phases:

a. Phase One: Computer modeling and analysis of the proposed design.

b. Phase Two: Evaluation of the proposed design.

c. Phase Three: Shop verification testing and evaluation of the assembled machine.

The API standards contain very specific, detailed information that may obscure the general scope or intent of the acceptance program described by the standard paragraphs. Two flow charts, Figures 1-33 and 1-34, have been developed to provide a global view of the acceptance program. Figure 1-33 contains a broad overview of the program; whereas, Figure 1-34 provides a very detailed flow chart of the process, complete with reference at each step to all applicable API standard paragraphs.

The importance of computer analysis may be understood by noting that calculated results are used to both accept the

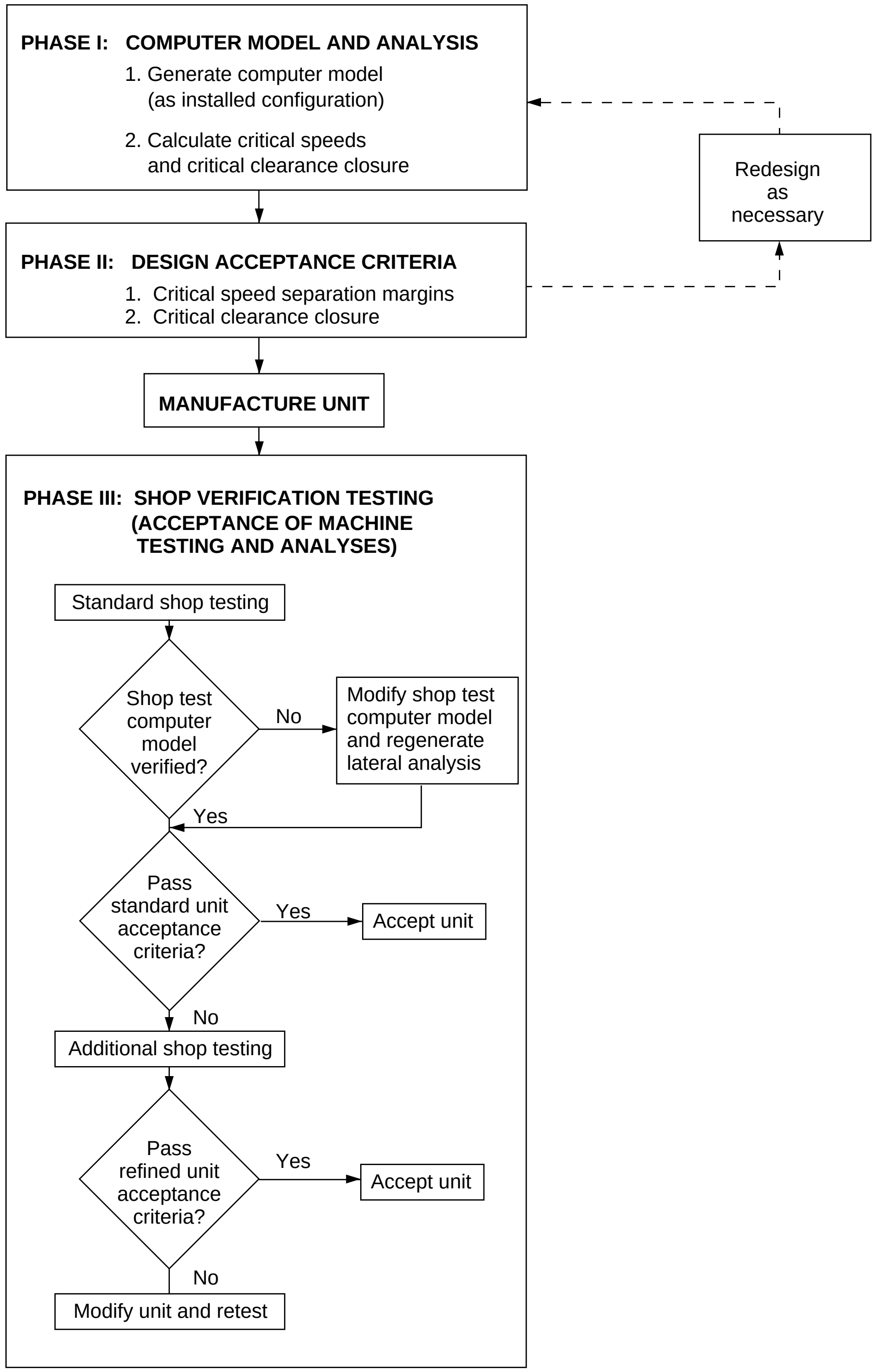


Figure 1-33—Three-Phase Vibration Acceptance Program
Outlined in API *Standard Paragraphs*

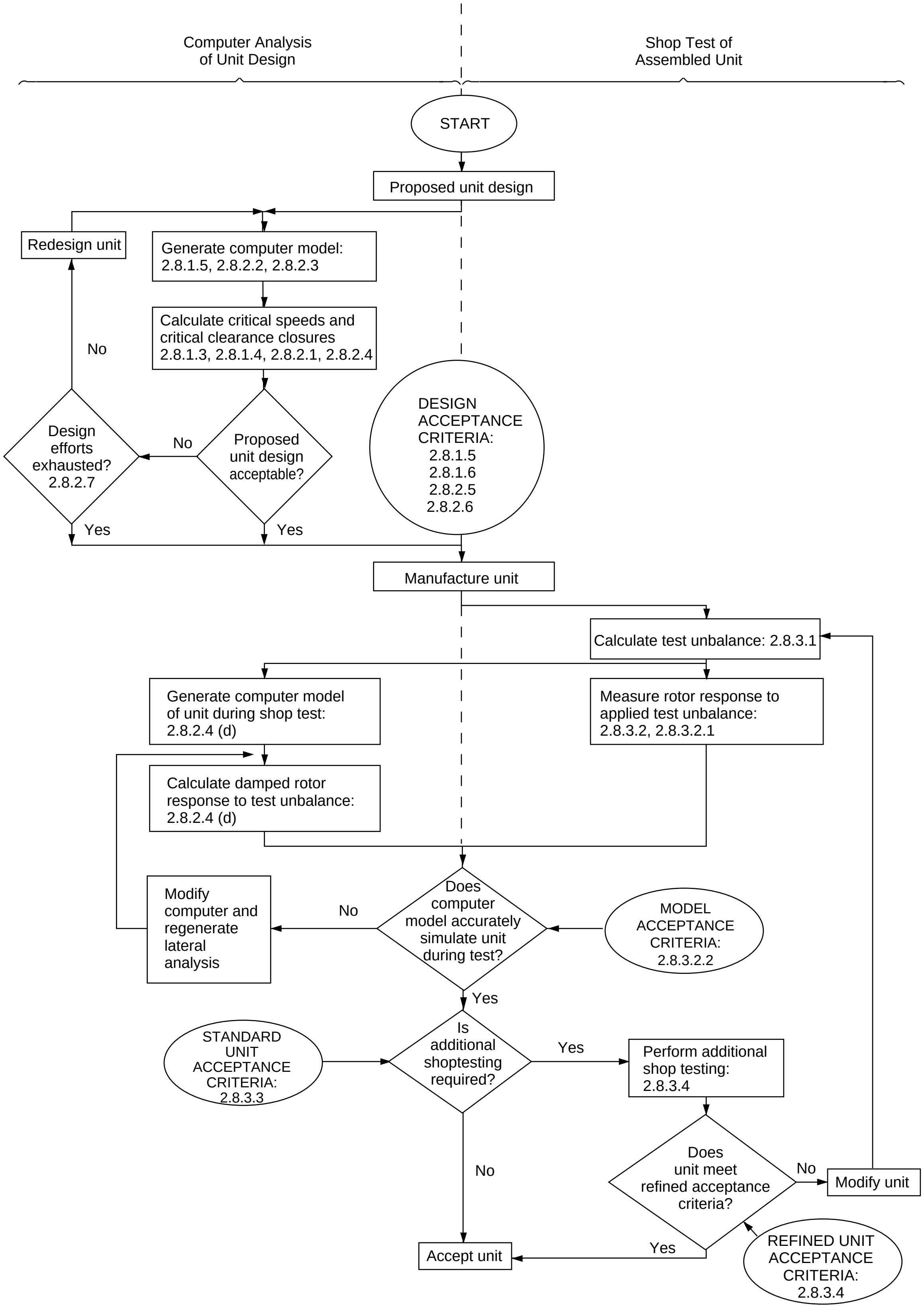


Figure 1-34—Detailed Flow Chart of API Vibration Acceptance Program
Outlined in API *Standard Paragraphs*

proposed design and interpret shop test measurements. Ideally, the design acceptance analysis is accomplished prior to release for manufacture. Once the proposed design is accepted by the purchaser and the unit is constructed, the standard paragraphs require that a mechanical run test be performed (see API *Standard Paragraphs*, 4.3.1.1 and 4.3.3). An important part of this test is the verification of the lateral analysis. This verification is accomplished by placing a test unbalance on the rotating element, measuring the rotor's response during run-up, and comparing the measurements with the results of the computer analysis. Assuming the unit's design is acceptable according to the design criteria found in Standard Paragraphs 2.8.2.5 and 2.8.2.6, if the lateral analysis agrees with the measurements recorded during the test within the tolerances specified in Standard Paragraph 2.8.3.2.2, then both the analysis and unit are accepted by the purchaser.

If, however, the unit does not meet the design criteria specified in Standard Paragraphs 2.8.2.5 and 2.8.2.6, then additional shop testing is required. Acceptance of the unit is then based on criteria outlined in Standard Paragraph 2.8.3.4. These criteria place restrictions on rotor vibration amplitudes at the bearings and seals (locations of critical clearance). Note that rotor vibrations are measured only at the displacement probes. All rotor vibrations at critical clearance locations are calculated by scaling measured displacements at the probes according to dynamic modeshapes calculated as part of the lateral analysis.

Such heavy reliance upon computer analysis requires that the model used to generate results be accurate. The note in Standard Paragraph 2.8.3.1 indicates that a separate unbalance response analysis may have to be performed specifically for the shop test if the test conditions (pressures, temperatures, speed, load, and so on) differ substantially from those encountered by the unit during normal operation in the field. If the results of the analysis performed for the shop test do not match the vibrations measured during the test within the tolerance specified in Standard Paragraph 2.8.3.2.2, then the shop test computer model must be modified until agreement between calculated and measured vibrations is obtained. Depending on the type and extent of the modifications to the shop test model, the computer model used to accept the design may also have to be corrected and the lateral analysis rerun. If the corrected computer model generates results that do not meet the design acceptance criteria outlined in Standard Paragraphs 2.8.2.5 and 2.8.2.6, then further shop testing is required to ensure that the unit meets the refined acceptance criteria outlined in Paragraph 2.8.3.4.

Finally, note that the API standard paragraphs provide for *generic* rotating equipment. Special considerations are important for particular types of machinery such as motors, gears, turbines, and others which are not covered in these standard paragraphs. The API specifications for specific machinery types should be referenced for more detailed requirements on particular types of units.

1.6.2 API *STANDARD PARAGRAPHS*

Note: Throughout this section the bolded text has been taken directly from the R-20 issue of the API *Standard Paragraphs* (see Appendix 1A). The bolded standard paragraphs will also have the letters SP preceding the paragraph number.

Material presented in Standard Paragraph 2.8.1 serves to standardize terminology and to provide basic definitions of the quantities used as evaluation criteria. For example, critical speeds and associated quantities (such as, amplification factor) are defined in this section. The definition of basic quantities is important because terms such as critical speeds have been previously defined in a number of different ways. For example, some turbomachine manufacturers have previously defined the *critical speeds* to be the rigid bearing resonance frequencies of the rotating assembly, because linearized bearing coefficients could not be accurately calculated. In general, the definitions presented in the API *Standard Paragraphs* reflect the perspective of the turbomachine user or plant operator.

SP 2.8 Dynamics

SP 2.8.1 CRITICAL SPEEDS

SP 2.8.1.1 When the frequency of a periodic forcing phenomenon (exciting frequency) applied to a rotor-bearing support system coincides with a natural frequency of that system, the system may be in a state of resonance.

System natural frequencies and forcing phenomena are discussed in 2.2 of this tutorial.

SP 2.8.1.2 A rotor-bearing support system in resonance will have its normal vibration displacement amplified. The magnitude of amplification and the rate of phase-angle change are related to the amount of damping in the system and the mode shape taken by the rotor.

The effect of damping on rotor vibrations is discussed in 2.2 of this tutorial.

SP Note: The mode shapes are commonly referred to as the first rigid (translatory or bouncing) mode, the second rigid (conical or rocking) mode, and the (first, second, third, ..., nth) bending mode.

Undamped mode shapes of the first three modes are provided in Figures 1-35 through 1-37 for an eight-stage steam turbine. These mode shapes display the characteristic bending seen in flexible shaft machines.

SP 2.8.1.3 When the rotor amplification factor (see Figure 1), as measured at the shaft radial vibration probes, is greater than or equal to 2.5, the corresponding frequency is called a critical speed, and the correspond-

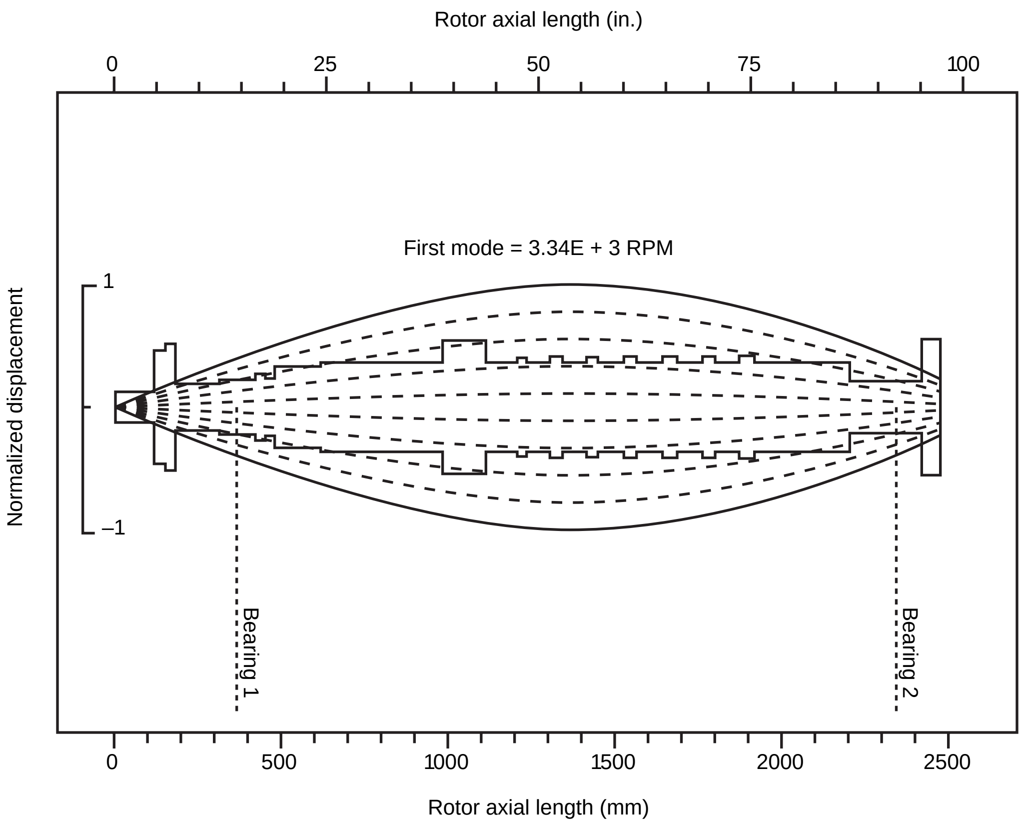


Figure 1-35—First Mode Shape for Eight-Stage Steam Turbine (Generated by Undamped Critical Speed Analysis)

ing shaft rotational frequency is also called a critical speed. For the purposes of this standard, a critically damped system is one in which the amplification factor is less than 2.5.

Note: (API *Standard Paragraphs* Figure 8 is reproduced in Figure 1-1 of this tutorial.)

Amplification factor is calculated using the half-power point method, as illustrated in Figure 1-1. API considers a mode of vibration with an amplification factor below 2.5 to be critically damped. These modes are not considered critical speeds because they generally do not result in high levels of rotor vibration. Unless the unit possesses a stiff shaft and operates with oil seals, the first critical speed will generally not be critically damped. Note, however, that many centrifugal compressors are designed to operate in close proximity to the second mode of vibration, as this mode is often critically damped.

SP 2.8.1.4 Critical speeds and their associated amplification factors shall be determined analytically by means of a damped unbalanced rotor response analysis and shall be confirmed during the running test and any specified optional tests.

The damped unbalanced response analysis is the primary means for calculating and evaluating rotor critical speeds because a critical speed is defined by API in terms of the calculated or measured response vibration data. The rotor dynamics analysis must be verified after the machine is built by comparing calculated results with actual machinery test vibration data.

SP 2.8.1.5 An exciting frequency may be less than, equal to, or greater than the rotational speed of the rotor. Potential exciting frequencies that are to be considered in the design of rotor-bearing systems shall include but are not limited to the following sources:

a. Unbalance in the rotor system.
b. Oil-film instabilities (whirl).
c. Internal rubs.
d. Blade, vane, nozzle, and diffuser passing frequencies.
e. Gear-tooth meshing and side bands.
f. Coupling misalignment.
g. Loose rotor-system components.
h. Hysteretic and friction whirl.

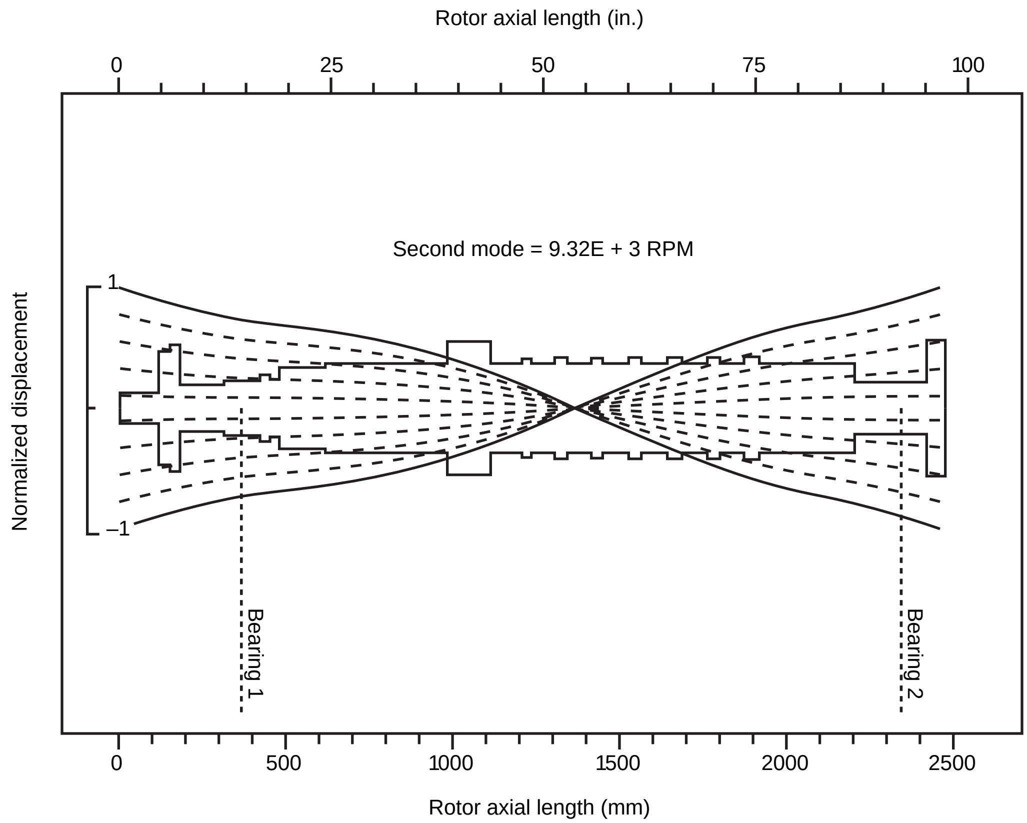


Figure 1-36—Second Mode Shape for Eight-Stage Steam Turbine (Generated by Undamped Critical Speed Analysis)

i. Boundary-layer flow separation.

j. Acoustic and aerodynamic cross-coupling forces.

k. Asynchronous whirl.

l. Ball and race frequencies of antifriction bearings.

The preceding list does not include all the potential sources of exciting frequencies, nor will all the sources listed be present for a given machine. This list is presented as a guide to the designer, suggesting that a variety of forces, both internal and external to the machine, must be identified and addressed as part of any complete rotor/bearing dynamics audit. The machine designer is responsible for identifying all potential excitation mechanisms that are relevant to a particular unit, and each mechanism must be properly considered in the design process. Table 1-10 presents some excitation mechanisms and associated frequencies that are typically encountered in turbomachinery.

Table 1-10—Typical Exciting Frequencies for Rotor/Bearing System

Source	Exciting Frequency
Unbalance	1 × rotor speed (N)
Oil film instability	0.4–0.45 × rotor speed with harmonics
Internal rubs	0.5 × rotor speed with half-harmonics
Blade, vane, nozzle, diffuser (passing)	Number of elements × rotor speed plus interference frequencies
Gear tooth meshing (and side bands)	Number of teeth × f(N) ± N, namely, F/N ± N ... NF ± N
Coupling misalignment	1,2,4 × rotor speed
Rotating mechanical looseness	1,2,3, ... , n × rotor speed
Hysteretic, friction whirl	Subsynchronous frequencies (typically 0.5 × rotor speed)
Boundary layer separation	Very low frequency subsynchronous (for example, 0.1 × rotor speed)
Aero cross coupling	0.4–0.5 × rotor speed
Asynchronous whirl	Subsynchronous frequencies
Ball/race frequencies of anti-friction bearings	Supersynchronous frequencies up to 10 × rotor speed

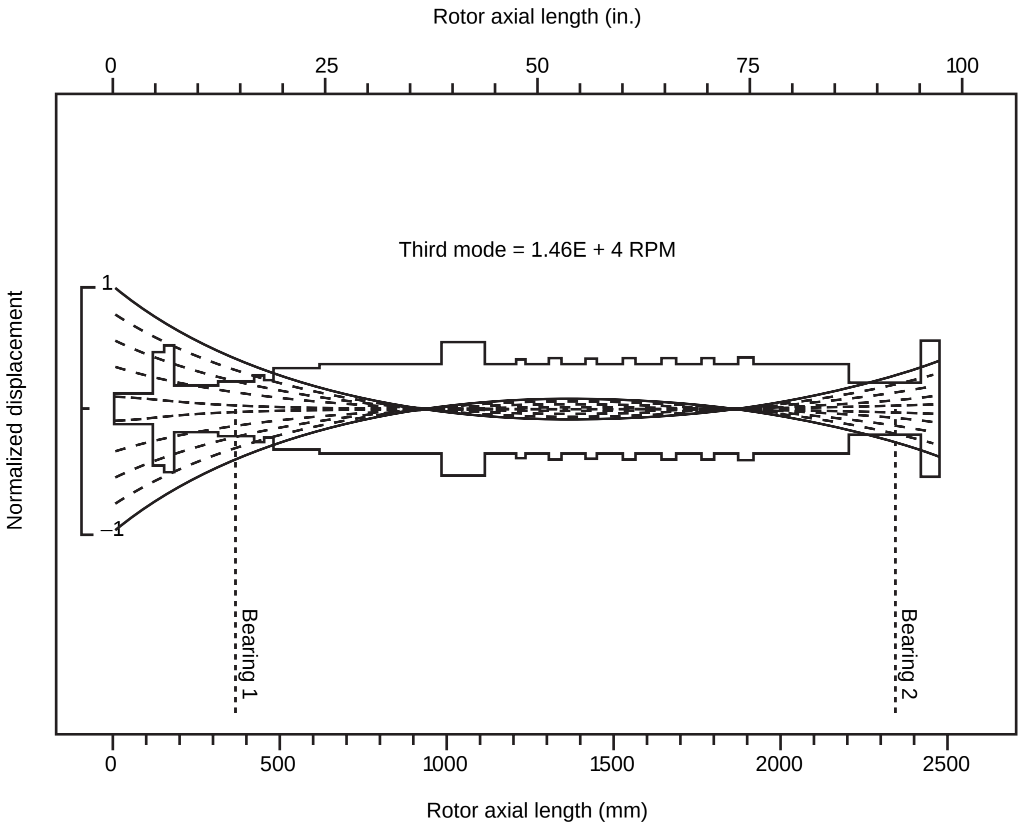


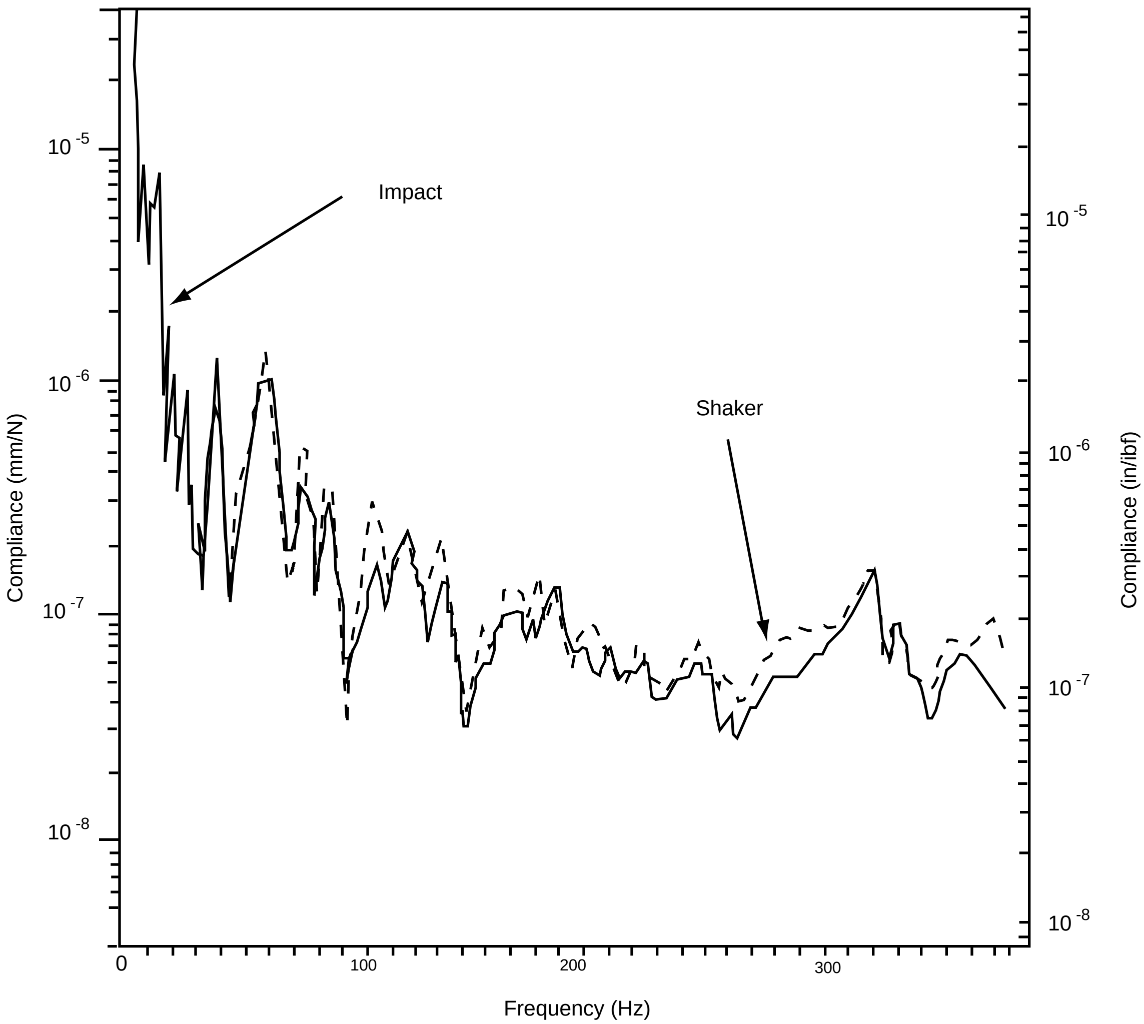
Figure 1-37—Third Mode Shape for Eight-Stage Steam Turbine (Generated by Undamped Critical Speed Analysis)

SP 2.8.1.6 Resonances of structural support systems may adversely affect the rotor vibration amplitude. Therefore, resonances of structural support systems that are within the vendor's scope of supply and that affect the rotor vibration amplitude shall not occur within the specified operating speed range or the specified separation margins (see 2.8.2.5) unless the resonances are critically damped.

All components and structures have natural resonant frequencies which may result in significant levels of vibration if a corresponding excitation mechanism exists. Some typical elements of turbomachinery that may have resonances of concern include bearing housings, oil drain lines, piping, pedestal supports, base plates, and foundations. If a support system structure has a resonance which is not adequately damped, the vibration response of this structure may be harmful to the machine and, in severe cases, may even prevent the safe and reliable operation of the machine. Such resonances must be located outside of the operating speed range unless they can be shown to be non-responsive (critically damped).

On electric motors, bearing housings may be supported by end plates. If the end plates are sufficiently thin, the effective radial stiffness of these supports is small, and problematic support resonances may be encountered. Problematic supports can be diagnosed by measuring frequency response data of the type displayed in Figure 1-38. This figure displays the measured compliance of a bearing support on the steam inlet end of a steam turbine. The bearing on this end of many steam turbines is attached to the sole plate or foundation using a thin flex plate that allows axial expansion of the unit during operation at temperature. Sharp peaks on the compliance plot with amplitudes several orders of magnitude greater than the average high frequency compliance would indicate the presence of significant support resonances.

SP 2.8.1.7 The vendor who is specified to have unit responsibility shall determine that the drive-train (turbine, gear, motor, and the like) critical speeds (rotor lateral, system torsional, blading modes, and the like) will not excite any critical speed of the machinery being supplied and that the entire train is suitable for the specified operating speed range, including any starting-speed detent



Note: Taken from Nicholas, et. al., "Improving Critical Speed Calculations Using Flexible Bearing Support FRF Compliance Data." (Courtesy of Turbomachinery Laboratory, Texas A&M University)

Figure 1-38—Example of Modal Testing Data: Measured Compliance of a Steam End Bearing Support

(hold-point) requirements of the train. A list of all undesirable speeds from zero to trip shall be submitted to the purchaser for his review and included in the instruction manual for his guidance (see Appendix C, Item 42).

Normally, when a multi-component equipment train is purchased, one of the equipment suppliers is assigned the responsibility for assembly coordination and performance of the entire train. This supplier is responsible for ensuring that all critical speeds related to the train are properly accounted for and will not degrade the intended operating envelope of the system. Typically, this supplier will provide a list of undesirable speeds in the form of individual or combined train Campbell diagrams, as illustrated in Figures 1-3 and 1-4. These diagrams define those speeds at which prolonged operation should be avoided because the associated vibration levels may lead to damage of the equipment.

According to API Standard 617, the centrifugal compressor manufacturer is responsible for torsional analysis of the train. Such responsibility includes directing motor, gear, coupling, and turbine manufacturers to modify proposed designs to meet torsional design requirements. Given this responsibility, the compressor manufacturer has generally been given the responsibility to ensure that all elements in the train possess adequate lateral rotor dynamics as well.

1.6.2.1 Phase I—Computer Model and Analysis (see Figure 1-30)

SP 2.8.2 LATERAL ANALYSIS [4.3.3.3.3]

In this section of the API *Standard Paragraphs*, the specific requirements placed on the lateral critical speed analysis are outlined. This section includes discussion of the types of analysis performed in order to evaluate the proposed design, the complicating effects that must be considered, and the manner in which the analysis is to be conducted. This section addresses the first two phases of the three-phase program (modeling and evaluation of a proposed design); if the analytical model of the proposed design is favorably evaluated using the criteria established here, then the machine purchaser releases the unit for manufacture.

SP 2.8.2.1 The vendor shall provide a damped unbalanced response analysis for each machine to assure acceptable amplitudes of vibration at any speed from zero to trip.

This statement simply reiterates the requirement that a damped unbalanced response analysis be conducted for a proposed design, as results generated by this analysis form the basis for evaluation of the unit's lateral vibration characteristics.

SP 2.8.2.2 The damped unbalanced response analysis shall include but shall not be limited to the following considerations:

Items a to e that follow essentially provide a manufacturer with the minimum requirements for an accurate unbalanced response analysis. This list is intended to provide a comprehensive, albeit not all-inclusive, outline of important modeling considerations. Note that additional specific requirements for the response analysis may exist for units of a particular type. Ultimately, it is the manufacturer's responsibility to determine and include all effects that are required to ensure the accuracy of the damped unbalance response analysis.

SP 2.8.2.2, a. Support (base, frame, and bearing-housing) stiffness, mass, and damping characteristics, including effects of rotational speed variation. The vendor shall state the assumed support system values and the basis for these values (for example, tests of identical rotor support systems, assumed values).

As noted in 2.2, bearing support characteristics can have a significant effect on calculated critical speeds, amplification factors, and so forth. This is particularly true when the unit operates near a support system's natural frequency. The general effect of flexible bearing supports or operation near a support resonance is to deprive the system of the damping generated by the bearings and thus adversely affect the unit's lateral characteristics.

In practice, most *hot* turbomachinery (for example, FCCU hot gas expanders, steam turbines, air compressors) possess a flexible bearing support that allows significant axial thermal growth. The radial stiffnesses of these supports (such as flex plates) are typically asymmetric and may be as low as 87.6 kilonewtons/millimeter (500,000 pounds/inch) in the horizontal direction. In order to generate an accurate lateral analysis, the effect of these supports must be included in the analysis of these units. Electric machinery with bearing housings supported by the end-plates must also be carefully modeled to account for bearing support flexibility. Inclusion of support flexibility may sometimes render unacceptable an otherwise sound rotor design.

The new API requirements call for the unit manufacturer to state the stiffness and damping characteristics of the support system and to inform the purchaser if these values are derived from measurements, calculations, or assumptions. Note that assumed values may be as valid or accurate as measured values. It is common practice for vendors to assume support stiffness and damping values based on experience with similar units. As previously mentioned, such *model tuning* often results in extremely accurate predictions of critical speeds and the associated amplification factors. The obvious drawback of such a procedure is that new unit design entails some risk. For new units, therefore, support characteristics may be estimated by performing a finite element stress analysis. The preferred and most accurate method of determining support properties for a given machine is to measure the frequency response function generated by a modal test. Figure 1-38 displays such data for the bearing support on the steam inlet end of a steam turbine. In practice, all three of the methods previously described are used by turbomachine manufacturers, depending on circumstances.

SP 2.8.2.2, b. Bearing lubricant-film stiffness and damping changes due to speed, load, preload, oil temperatures, accumulated assembly tolerances, and maximum to minimum clearances.

As discussed in the section on bearing modeling, a variety of factors can influence the stiffness and damping characteristics of a bearing design. The analyst must account for these effects in the analysis by performing a lateral rotor dynamics analysis for the two bearing clearance cases that will generate maximum and minimum calculated bearing stiffnesses. In this manner, the full variation in the lateral response of the unit resulting from manufacturing tolerance in the bearing clearance will be determined. The maximum bearing stiffnesses generally occur at minimum clearance; whereas, the minimum bearing stiffnesses generally occur at maximum clearance. All other parameters associated with the bearings may generally be fixed at the nominal or expected values. For example, in tilting pad journal bearings, the ground pad clearance may be set at its nominal value, the average of the minimum and maximum dimensions.

SP 2.8.2.2:
c. Rotational speed, including the various starting-speed detents, operating speed and load ranges (including agreed-upon test conditions if different from those specified), trip speed, and coast-down conditions.
d. Rotor masses, including the mass moment of coupling halves, stiffness, and damping effects (for example, accumulated fit tolerances, fluid stiffening and damping, and frame and casing effects).
e. Asymmetrical loading (for example, partial arc admission, gear forces, side streams, and eccentric clearances).

Asymmetrical loading is particularly important in certain classes of rotating equipment. For gears, the radial bearing loadings generated by power transmission at the mesh are significant and must be accurately evaluated. For steam turbines, the inlet nozzles often do not cover a full 360 degrees, resulting in partial arc steam admission. Partial arc steam admission creates a static load on a turbine rotor in a direction perpendicular to the plane formed by the rotor centerline and the midpoint of the nozzle admission arc. When a turbine possesses only one or two stages or if the first stage generates a majority of the turbine's power, then partial arc steam admission effects may double or triple the journal static loading. The increased journal static loading may have a profound effect on the linearized bearing coefficients and, consequently, may significantly affect calculated rotor dynamics characteristics. Note that partial arc steam admission effects should not be confused with the aerodynamic forces that are generated by interaction between the blade tips and the blade tip seals and cause *steam whirl*.

SP 2.8.2.2, f. The influence, over the operating range, of the calculated values for hydrodynamic stiffness and damping generated by the casing end seals.

As noted in 3.3 of this tutorial, floating ring oil seals in centrifugal compressors often act like radial bearings and affect the unit's lateral rotor dynamic response, particularly when the sealing pressure is greater than 3.447 megapascals (500 pound-force/inches^2). This item requires that the effect of fluid film seals be considered in the unbalanced response analysis. The seal analysis should also account for any difference between job seals and test seals if special, high clearance seal rings are used during the shop mechanical acceptance tests. Test seals will not have the same influence on response as the actual job seals because of differences in the clearances and operating conditions. These differences can have a major influence on the response characteristics of the unit. Some units that successfully passed shop mechanical run tests have failed to properly operate after installation in the field because the increased destabilizing forces generated by the job seals had a deleterious effect on the stability of the compressor during normal operation.

SP 2.8.2.2 (continued) For machines equipped with antifriction bearings, the vendor shall state the bearing stiffness and damping values used for the analysis and either the basis for these values or the assumptions made in calculating the values.

Figure 1-39 shows a characteristic antifriction bearing design. Vendor stiffness values for antifriction bearings should be based on the material change due to elastic deflection for the balls as well as inner and outer races. These yield values are dependent on radial load, thrust load, and bearing assembly axial preload. Damping values are usually based on material hysteresis effects only and are, therefore, very small, approximately 1 to 2 percent of critical damping. Because these stiffness and damping characteristics can be difficult to accurately quantify, great care must be exercised in modeling and analyzing antifriction bearing systems.

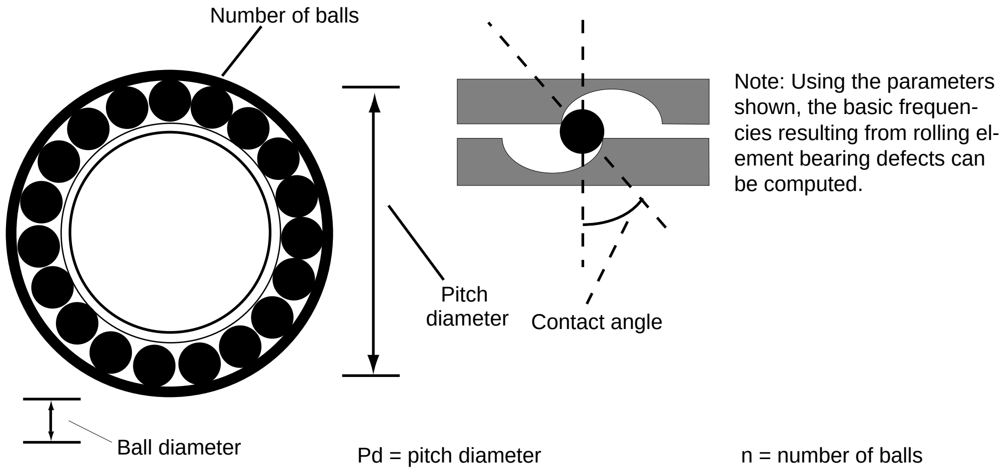
Figure 1-40 displays a comparison between the first mode responses of a rotor supported by fluid-film bearings and antifriction bearings. The decrease in damping provided by the antifriction bearings relative to the fluid film bearings results in critical speeds that are more highly amplified. The unit whose synchronous response is displayed in Figure 1-40 operates successfully on antifriction bearings, but only after a squeeze film damper was added between the bearings and the bearing supports to provide increased damping.

SP 2.8.2.3 When specified, the effects of other equipment in the train shall be included in the damped unbalanced response analysis (that is, a train lateral analysis shall be performed).

Note: This analysis should be considered for machinery trains with coupling spacers greater than 1 meter (36 inches), rigid couplings, or both.

This paragraph is intended to address specific cases where standard coupling modeling procedures (coupling half-weight lumped at the coupling center of gravity (CG) do not result in adequate prediction of the rotor's lateral dynamic behavior. It must be noted that a coupled train lateral analysis is rarely employed to determine the lateral characteristics of train elements. A coupled lateral analysis is necessary, however, for the two distinct cases mentioned above: long couplings (DBSE greater than 914.4 millimeters or 36 inches) and rigid couplings.

Long couplings are most often employed as load and auxiliary drive couplings in gas turbine applications. These couplings may operate above their first lateral critical speed, and the train lateral analysis will help gauge the importance of this condition. Rigid couplings are used when flexible element or gear type couplings simply cannot withstand the drive torques necessary to operate the equipment. These couplings transmit both shear and moment so rigidly coupled that trains respond dynamically as a single multi-bearing ($N_{bearings} > 2$) machine rather than as separate, uncoupled components. A coupled train lateral analysis is necessary to



Pd = pitch diameter
Bd = ball diameter
n = number of balls
ϕ = contact angle

BEARING CHARACTERISTIC FREQUENCIES:

Defect on outer race (Ball pass frequency outer)	$\frac{n}{2}\,\frac{RPM}{60}\left(1 - \frac{Bd}{Pd}\cos\phi\right)$	(1)
Defect on inner race (Ball pass frequency inner)	$\frac{n}{2}\,\frac{RPM}{60}\left(1 + \frac{Bd}{Pd}\cos\phi\right)$	(2)
Ball defect (ball spin frequency)	$\frac{Pd}{2Bd}\,\frac{RPM}{60}\left[1 - \left(\frac{Bd}{Pd}\right)^2 \cos^2\phi\right]$	(3)
Fundamental train frequency	$\frac{1}{2}\,\frac{RPM}{60}\left(1 - \frac{Bd}{Pd}\cos\phi\right)$	(4)

Figure 1-39—Anti-Friction Bearing Design Characteristics

identify the lateral characteristics of such trains. The principal example of such a train is a large steam-turbine driven generator train with the generator's rated power exceeding 100 megawatts (134,100 horsepower). While rigid couplings are commonly found in the power generation industry, they are rarely needed in critical plant equipment because the power requirements are not as large. Rigid couplings may also be found in reciprocating compressors, but such equipment is beyond the scope of this document.

The vast majority of critical petroleum plant turbomachinery are designed to allow flexible couplings to transmit the drive torque between units. These couplings effectively attenuate transmitted moments over a large range of alignments and serve to isolate the train components from each other. Except for the two specific cases discussed above, it is sufficiently accurate to model and analyze rotors as individual, uncoupled machines.

SP 2.8.2.4 As a minimum, the damped unbalanced response analysis shall include the following:

a. A plot and identification of the mode shape at each resonant speed (critically damped or not) from zero to trip, as well as the next mode occurring above the trip speed.

This paragraph requires mode shapes be provided for all criticals inclusive of the mode above the unit's trip speed. As most flexible shaft rotating equipment in the petrochemical industry operates between the first and second critical speeds, at least two mode shapes are required for the lateral analysis. Refer to Figure 1-6 for the three lowest frequency *idealized* mode shapes for beam-type rotors and to Figures 1-35 through 1-37 for mode shapes calculated for an eight-stage steam turbine.

Mode shapes are valuable tools to both the designer and the purchaser because they provide information about the rotor's sensitivity to unbalance at various speeds. For example, if the response sensitivity of a unit's first mode to coupling unbalance is small, then field balancing on the coupling hub to correct excessive vibrations near the first critical speed will likely prove futile. Mode shapes are also used during the

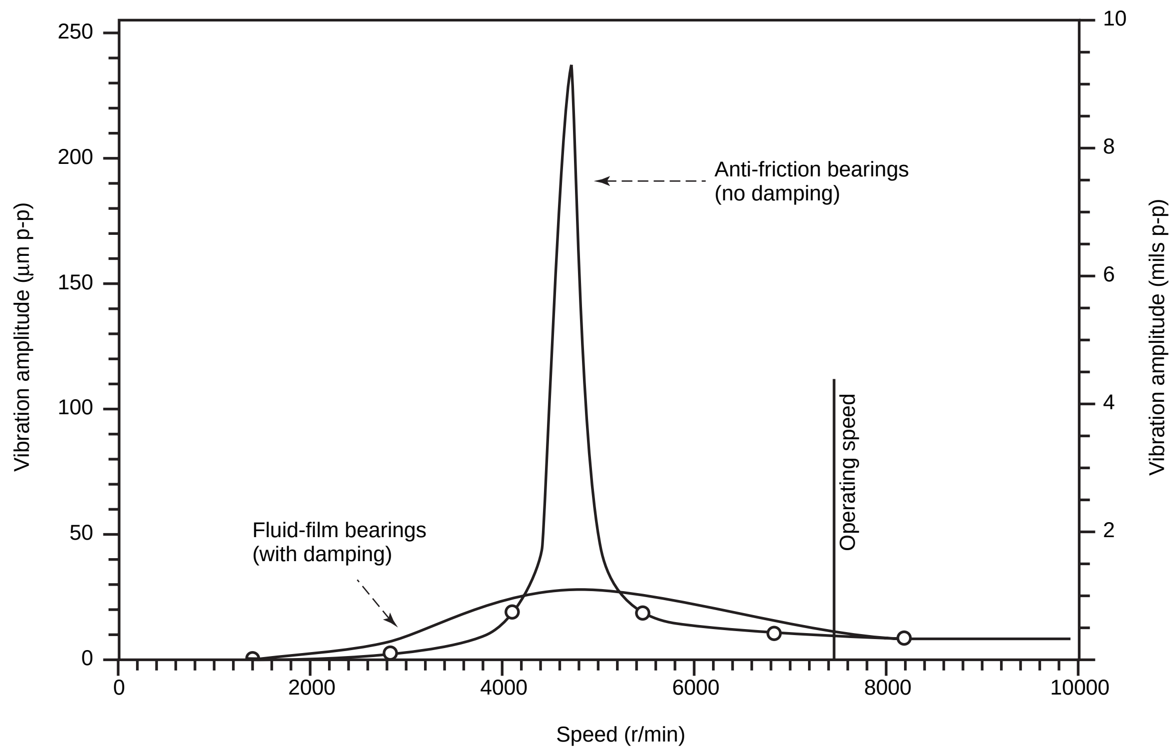


Figure 1-40—Comparison of First Mode Response With Fluid-Film Bearings and Anti-Friction Bearings

lateral analysis to determine where unbalances should be located to excite critical speeds of concern to the purchaser. In addition, knowledge of the mode shapes permits estimation of rotor displacements at bearings and seals if given the rotor's response at probe locations.

If, in addition to all criticals located between zero speed and the unit's trip speed, the first critical above trip speed is calculated, then the response analysis will indicate if the proposed machine is operating immediately below a responsive lateral critical. Operation of the unit immediately below a critical is undesirable for the following reasons:

a. The machine may be severely damaged in the event of an accidental overspeed caused by a sudden loss of load or a governor failure.
b. The frequency of the critical speed may drop down into the unit's operating speed range if the bearing clearances increase over time.

SP 2.8.2.4, b. Frequency, phase, and response amplitude data (Bodé plots) at the vibration probe locations through the range of each critical speed, using the following arrangement of unbalance for the particular mode. This unbalance shall be sufficient to raise the displacement of the rotor at the probe locations to the vibration limit defined by the following equation:

In SI units,

$$L_v = 25.4\sqrt{\frac{12{,}000}{N}} \qquad \textbf{(1)}$$

In US Customary units,

$$L_v = \sqrt{\frac{12{,}000}{N}}$$

Where:

L_v = vibration limit (amplitude of unfiltered vibration), in micrometers (mils) peak to peak.
N = operating speed nearest the critical of concern, in revolutions per minute.

Frequency, phase, and response amplitude data are typically presented graphically in Bodé plots. Figure 1-2 displays a Bodé plot for the midspan response of an eight-stage steam turbine.

Figure 1-41 compares amplitude-only Bodé plots generated using calculated and measured vibration data for the steam turbine.

Several damped unbalance response cases must be analyzed in order to satisfy the requirements of Standard Para-

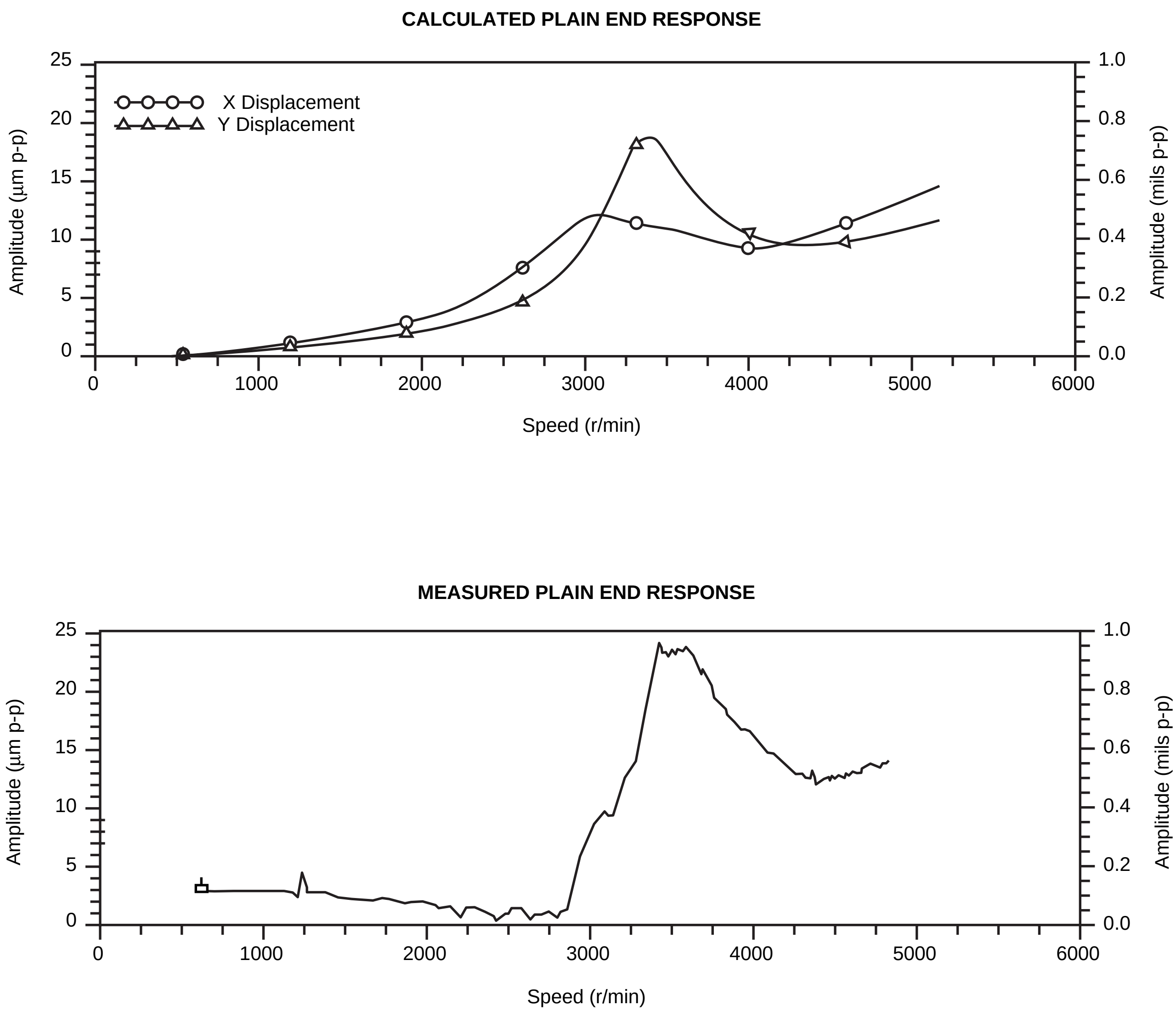


Figure 1-41—Comparison of Calculated and Measured Unbalanced Responses Eight-Stage 12 MW (16,000 HP) Steam Turbine

graph 2.8.2.4, Item b. The number of unbalance cases that appear in the final lateral dynamics report issued by the equipment manufacturer is dependent upon the locations of the critical speeds relative to the normal operating speed range of the unit. Two cases must be considered:

a. The unit is designed to operate below the first critical speed.
b. The unit is designed to operate between two critical speeds (usually the first and second critical speeds).

Machinery that is typically designed to operate below the first critical includes all flexibly-coupled electric machinery and speed-changing gearboxes under load. Figure 1-42 provides a typical Bodé plot for a constant speed two-pole motor. This motor is designed to operate below the first critical speed (3700 revolutions per minute). In this example, the *critical of concern* is the first critical speed. One unbalance case must be considered to fulfill the requirements of 2.8.2.4, Item b. Sufficient unbalance must be applied to the computer model of the rotor to raise the displacement of the rotor at the probe locations to the following level.

$$L_v = 25.4\sqrt{\frac{12{,}000}{N}}$$

$$L_v = 46.7 \text{ micrometers peak-to-peak (pp) (1.83 mils pp)}$$

Most steam turbines and centrifugal compressors are generally designed to operate above the first critical speed, sometimes above the first two critical speeds. Figure 1-43

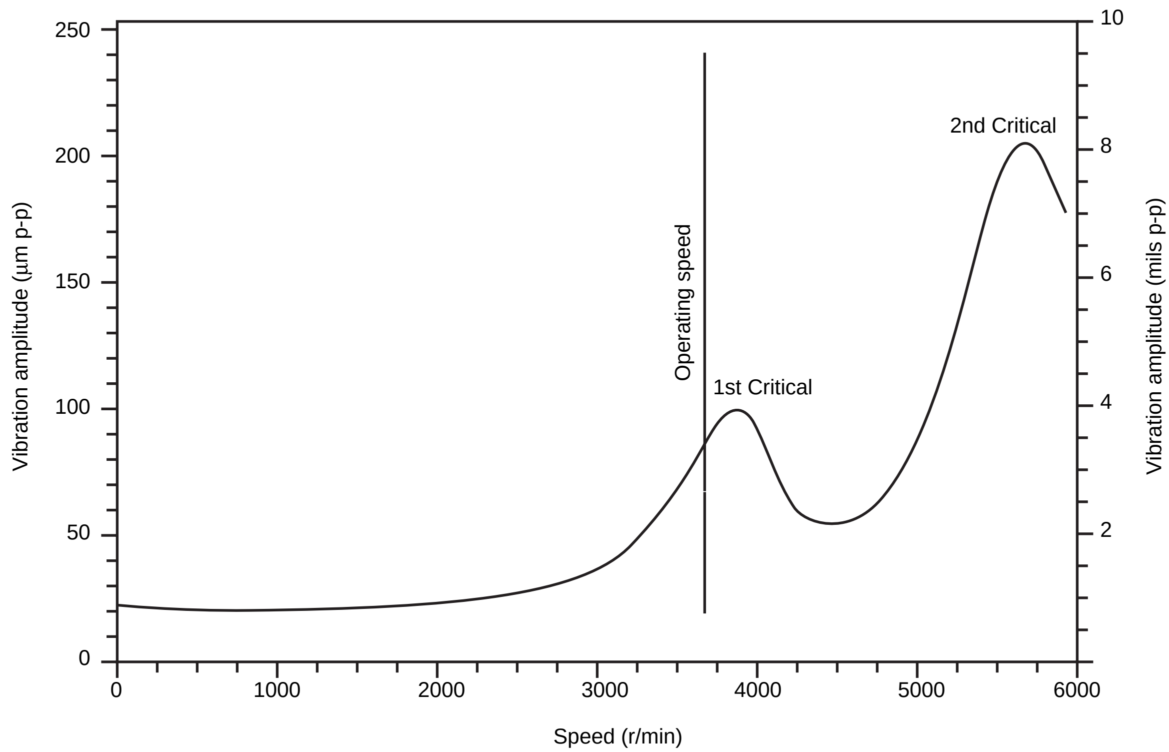


Figure 1-42—Response of a Constant Speed, Two-Pole Motor

provides a typical Bodé plot for variable speed steam turbine. This turbine is designed to operate between the first and second critical speeds. In this example, there are two criticals of concern, and two unbalance cases must be considered:

a. One critical of concern is the first critical speed, and the operating speed nearest this critical is 5000 revolutions per minute (possibly the minimum governor speed). Sufficient unbalance must be applied to the computer model of the rotor to raise the displacement of the rotor at the probe locations to the following level.

$$L_v = 25.4\sqrt{\frac{12,000}{N}}$$

$$L_v = 39.3 \text{ micrometers pp (1.55 mils pp)}$$

b. The second critical of concern is the second critical speed, and the operating speed nearest this critical is 7000 revolutions per minute maximum continuous operating speed (MCOS). Sufficient unbalance must be applied to the computer model of the rotor to raise the displacement of the rotor at the probe locations to the following level.

$$L_v = 25.4\sqrt{\frac{12,000}{N}}$$

$$L_v = 33.3 \text{ micrometers pp (1.31 mils pp)}$$

Given the preceding discussion, a general definition of the term *criticals of concern* is offered: The *criticals of concern* are all the critical speeds located below the maximum operating speed of a unit plus the critical speed located immediately above the maximum operating speed of the unit.

The criticals of concern are typically identified by performing an unbalance response analysis with an unbalance distribution designed to simultaneously excite the lowest three or four critical speeds. A total unbalance of $4W/N$ with the following distribution on the rotor are placed for the purpose of identifying critical speeds of concern:

a. Two $2W/3N$ unbalances 180 degrees out of phase are placed at the rotor-end planes.
b. One $8W/3N$ unbalance 90 degrees out of the plane formed by the end plane unbalances is placed at the midpoint between the two bearings.

Note: In the above discussion,

W = Total weight of the rotating element, in pounds.
N = Maximum continuous operating speed of the rotor, in revolutions per minute.

These variable definitions differ from those presented by API in the API *Standard Paragraphs*.

The response analysis performed for the unbalance distribution just described can be used to estimate the actual unbalance needed to raise the vibration limit at the probes to the specified amount. If one notes that the unbalance re-

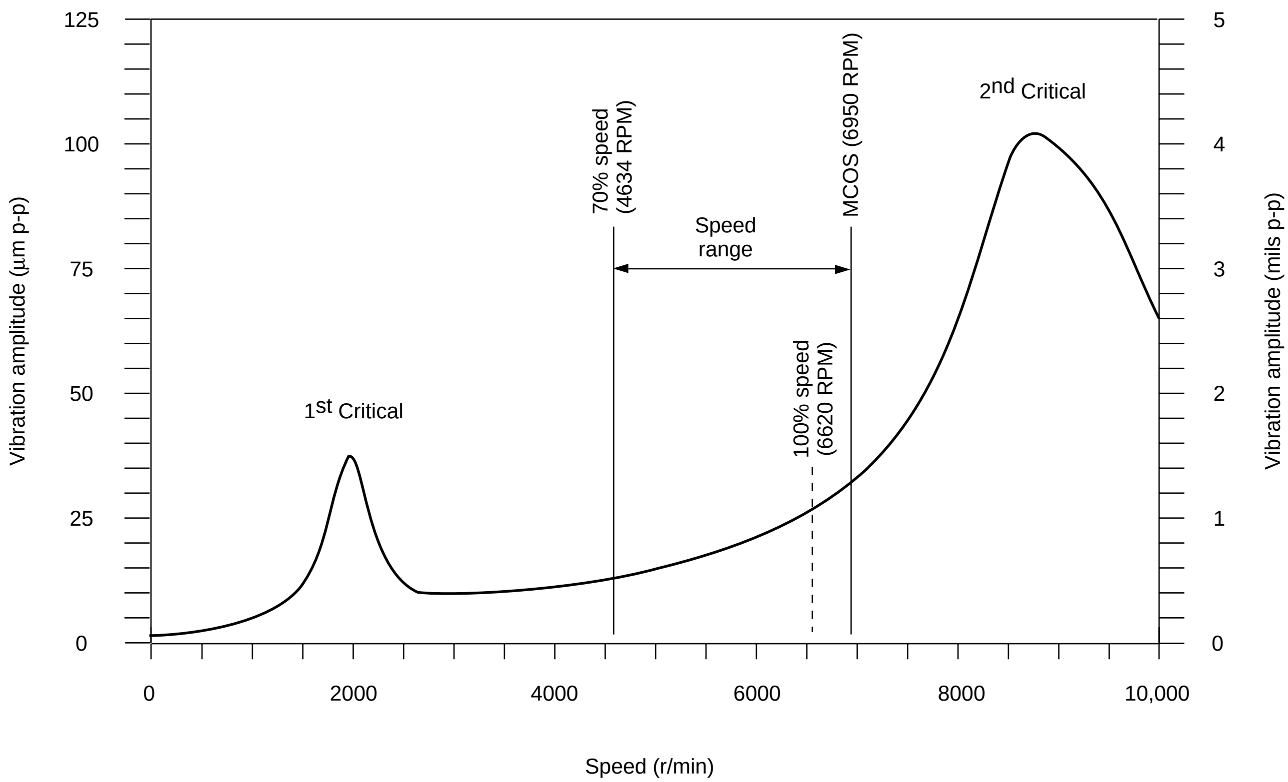


Figure 1-43—Response of a Variable Speed Steam Turbine

sponse analysis is linear, then it follows that the required unbalances can be scaled according to the calculated response sensitivity of the rotor to the applied unbalance at the probe locations.

SP 2.8.2.4, Item b. (continued) This unbalance shall be no less than two times the unbalance defined by the following equation:

In SI units,

$$U = 6350W/N$$

(2)

In US Customary units,

$$U = 4W/N$$

Where:

- ***U*** **= input unbalance from the rotor dynamic response analysis, in gram-millimeters (ounce-inches).**
- ***W*** **= journal static weight load, in kilograms (pounds), or for bending modes where the maximum deflection occurs at the shaft ends, the overhung weight load (that is, the weight outboard of the bearing), in kilograms (pounds).**

The journal static weights are equal to the magnitude of the bearing reactions when no external forces are placed on the rotor.

- ***N*** **= operating speed nearest the critical of concern, in revolutions per minute.**

The amount of unbalance defined by the equation for *U* above is the total residual unbalance that the newly constructed rotor is allowed to have after balancing. It is not uncommon, however, for operating rotors in the field to have levels of unbalance more than twice the API allowable residual unbalance. Therefore, the design must permit operation with the unbalance specified in the preceding without exceeding the allowable vibration level.

At this point, the amount of unbalance to be applied to the computer model of the rotor has been thoroughly discussed. Guidance on placing the unbalance on the rotor is given below.

SP 2.8.2.4, Item b. (continued) The unbalance weight or weights shall be placed at the locations that have been analytically determined to affect the particular mode most adversely. For translatory modes, the unbalance shall be based on both journal static weights and shall be applied at the locations of maximum displace-

ment. For conical modes, each unbalance shall be based on the journal weight and shall be applied at the location of maximum displacement of the mode nearest the journal used for the unbalance calculation, 180 degrees out of phase. Figure 2 shows the typical mode shapes and indicates the location and definition of *U* for each of the shapes. [2.8.2.4, Item d; 2.8.2.5; 2.8.2.6; 2.8.3.1, Item b; 2.8.3.4, Item b]

Note: API Standard Paragraphs Figure 2 is reproduced herein as Figure 1-44.)

The mode shapes required in Standard Paragraph 2.8.2.4, Item a, standardize locations of applied unbalances. The calculated mode shapes are compared to those presented in Figure 1-44, and the appropriate unbalance distribution is established for each mode. The point of greatest sensitivity to unbalance for a particular mode is the point of greatest displacement on the mode shape.

For the motor unbalance case discussed above, there is only one critical of concern. This critical is most adversely affected or *excited* by placing the unbalance at the point of greatest displacement in the mode shape associated with the motor's first critical speed. For beam rotors, this point is about midway between the two bearings.

When there is more than one critical of concern, different unbalance distributions must be used to ensure that the criticals are properly excited. For the eight-stage steam turbine examined above, the first and second critical speeds are the criticals of concern. The first critical is excited by placing a single unbalance at the point of greatest displacement in the mode shape associated with the first critical. Again, for beam rotors, this point is usually about midway between the two bearings. The second critical is excited by placing unbalance at the two points of greatest modal displacement 180 degrees out-of-phase. It is not uncommon for the points of greatest displacement on the mode shape associated with the second critical to occur at the shaft end planes. When this occurs, the *W* in the 4*W*/*N* unbalance limit refers not to the journal static weight, but to the weight of the rotating assembly outboard of the bearing.

SP 2.8.2.4, c. Modal diagrams for each response in Item b above, indicating the phase and major-axis amplitude at each coupling engagement plane, the centerlines of the bearings, the locations of the vibration probes, and each seal area throughout the machine. The minimum design diametral running clearance of the seals shall also be indicated. [2.8.3.4]

A modal diagram with all the required information superimposed on it should be generated from the analysis results using the correct amount and placement of unbalance weight, for each critical speed from zero to trip speed and the first critical above trip speed. Note that running clearances should be displayed in the modal diagrams, not the static clearances measured when the rotor is stationary. Figure 1-45 indicates the important design information that should be shown graphically on these mode plots.

The modal drawings from this analysis will be used during the mechanical run test to determine the shaft internal and coupling plane displacements by measuring the shaft radial vibration at the probes. These values are then ratioed according to the vibration levels provided on the modal diagram to determine the actual vibration levels at critical clearance locations.

SP 2.8.2.4, d. A verification test of the rotor unbalance is required at the completion of the mechanical running test to establish the validity of the analytical model. Therefore, additional plots based on the actual unbalance to be used during this test shall be provided as follows: For machines that meet the requirements of 2.8.2.4, Item b, and 2.8.2.5, additional Bodé plots, as specified in 2.8.2.4, Item b, shall be provided. The location of the test unbalance shall be determined by the vendor. The amount of unbalance shall be sufficient to raise the vibration levels, as measured at the vibration probes, to those specified in 2.8.2.4, Item b. In all cases, the unbalance plots shall include the effects of any test-stand conditions (including the effects of test seals) that may be used during the verification test of the rotor unbalance (see 2.8.3). [2.8.3.1; 2.8.3.2]

Once the proposed design meets the criteria specified in Standard Paragraphs 2.8.2.5 and 2.8.2.6, or the purchaser grants the manufacturer a special waiver from these criteria, then a damped unbalance response analysis must be conducted with the unit in its shop test configuration. The analysis required by this paragraph is used to verify the analysis conducted for the unit's design as it operates in the field. The effect of all test-stand operating conditions such as temperature, pressure, and load as well as the effects of test oil seals should be considered in the shop test model. Additionally, the size and location of the unbalance applied to the computer model must be the same as that proposed for the unit during the shop mechanical run test.

Refer to Figure 1-34 for a flow chart that details the interrelationship of the computational analysis with the shop test measurements.

SP 2.8.2.4, e. Unless otherwise specified, a stiffness map of the undamped rotor response from which the damped unbalanced response analysis specified in Item c above was derived. This plot shall show frequency versus support system stiffness, with the calculated support system stiffness curves superimposed.

A stiffness map, also referred to as an *undamped critical speed map*, graphically displays the effect of a parametric variation of bearing stiffness on calculated undamped critical speeds. Figure 1-46 displays an undamped critical speed

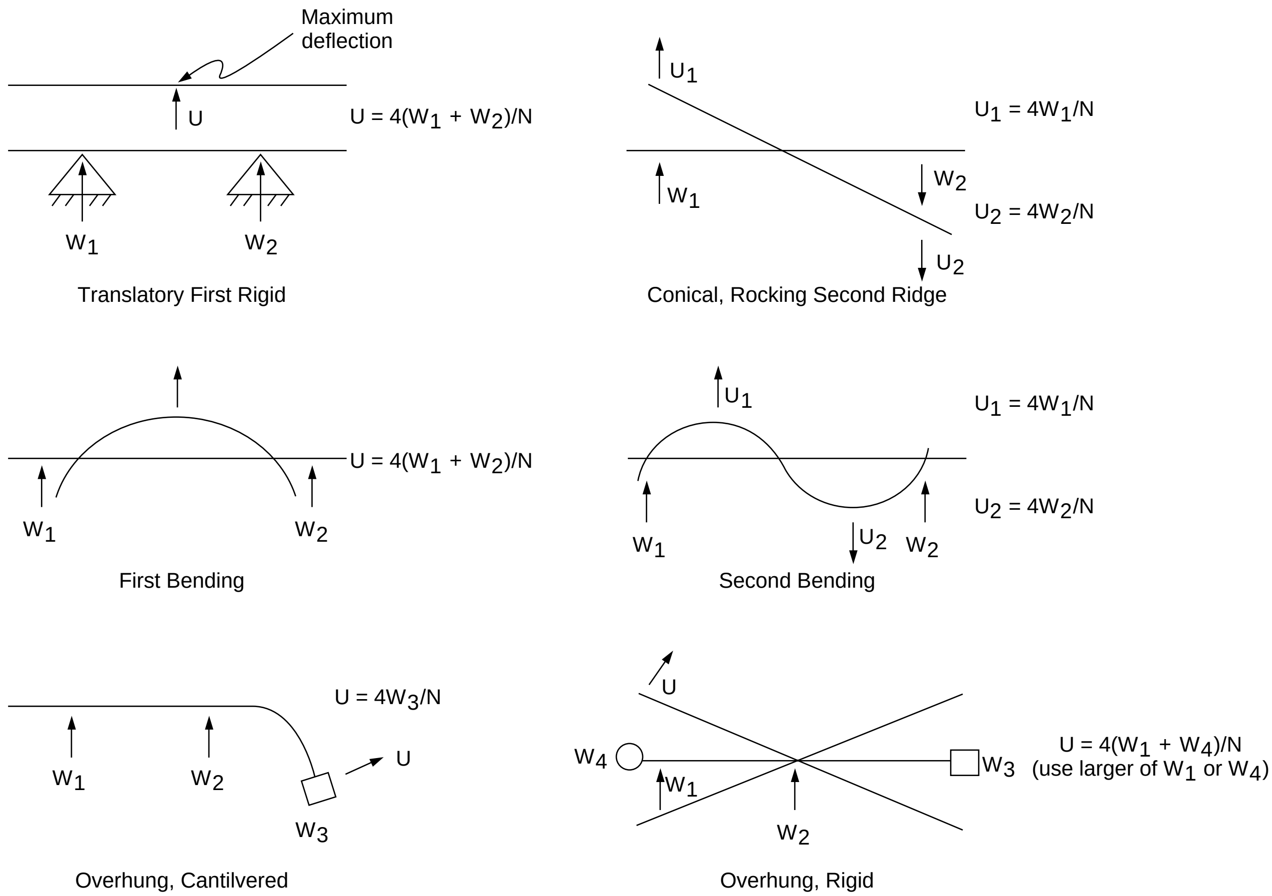


Figure 1-44—Unbalance Calculations and Placements in Figure 2 of API *Standard Paragraphs*

map generated for an eight-stage steam turbine. A general discussion of critical speed maps has been provided in 2.3 of this tutorial. This graph can be a useful tool in evaluating the general effect of bearing modifications or retrofits on a unit's lateral dynamic characteristics. For example, the cross-plotted bearing stiffnesses that appear in Figure 1-46 represent the principal stiffnesses of a 5-pad load-between-pads tilting pad bearing. When the turbine originally operated with stiffer 4-axial groove sleeve bearings, the unit suffered from an interference of the first critical speed with minimum operating speed. Installation of the lower stiffness tilting pad bearings decreased the location of the first critical and allowed the unit to operate with a significantly enhanced margin at minimum operating speed.

As previously noted, the critical speed map is limited to circular rotor response with no provision for such effects as bearing damping, seal damping, cross-coupling, unbalance effects, and others. Therefore, the critical speed map provides a simplified representation of the system and generally does not provide accurate prediction of the unit's actual critical speeds. *For this reason, the stiffness map should not be used to calculate critical speeds defined in Standard Paragraph 2.8.1.3.* According to Standard Paragraph 2.8.1.4, critical speeds can only be calculated using the damped unbalanced response analysis. A critical speed map is properly used to determine the general characteristics of a unit (high critical amplification and so forth) and to assist in determining the influence of the bearings on the rotor dynamic characteristics of the unit.

SP 2.8.2.4, f. For machines whose bearing support system stiffness values are less than or equal to 3.5 times the bearing stiffness values, the calculated frequency-dependent support stiffness and damping values (impedances) or the values derived from modal testing. The results of the damped unbalanced response analysis shall include Bodé plots that compare absolute shaft motion with shaft motion relative to the bearing housing. [2.8.3.1]

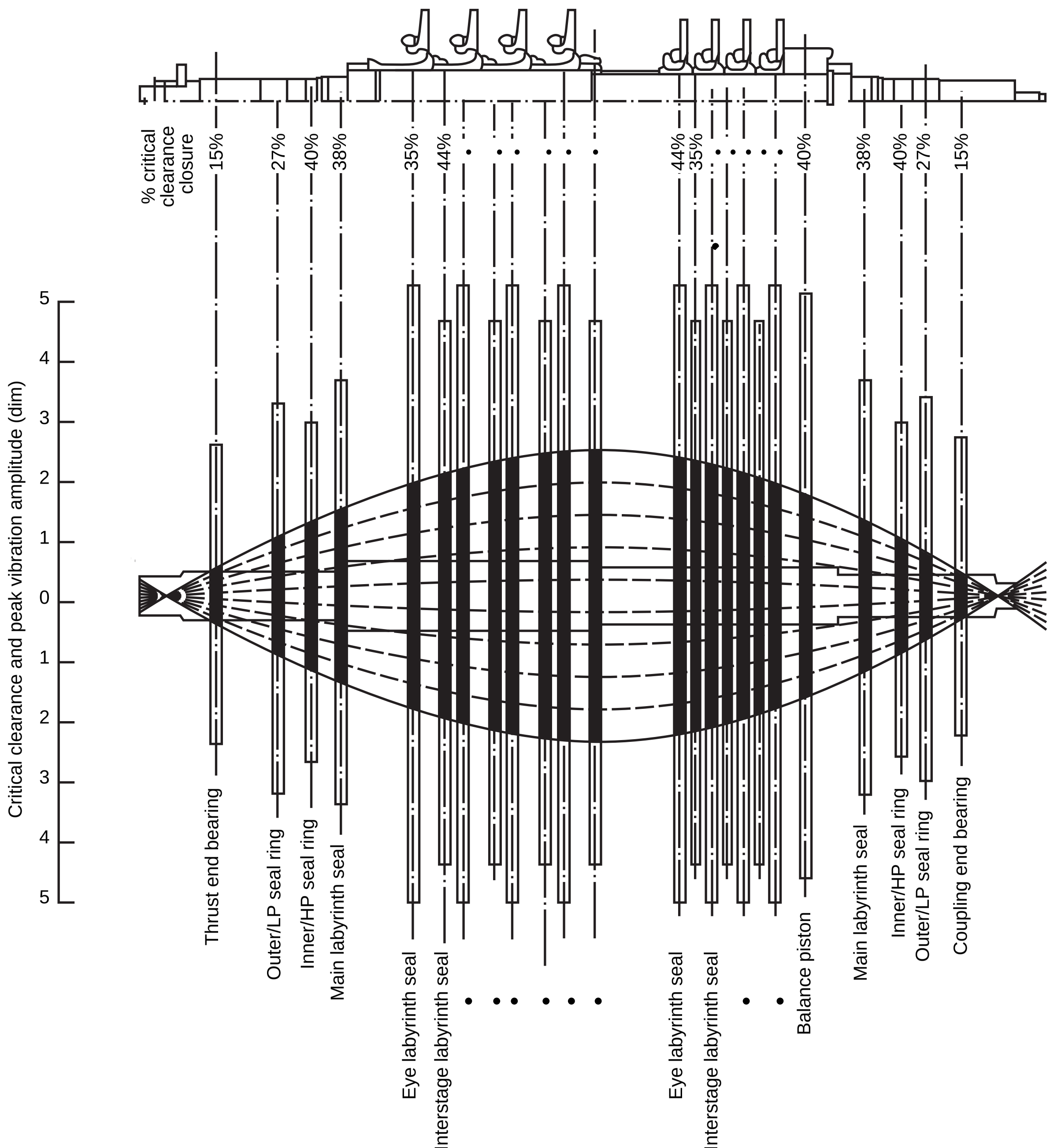


Figure 1-45—Modal Diagram Showing Critical Clearances and Peak Vibration Levels

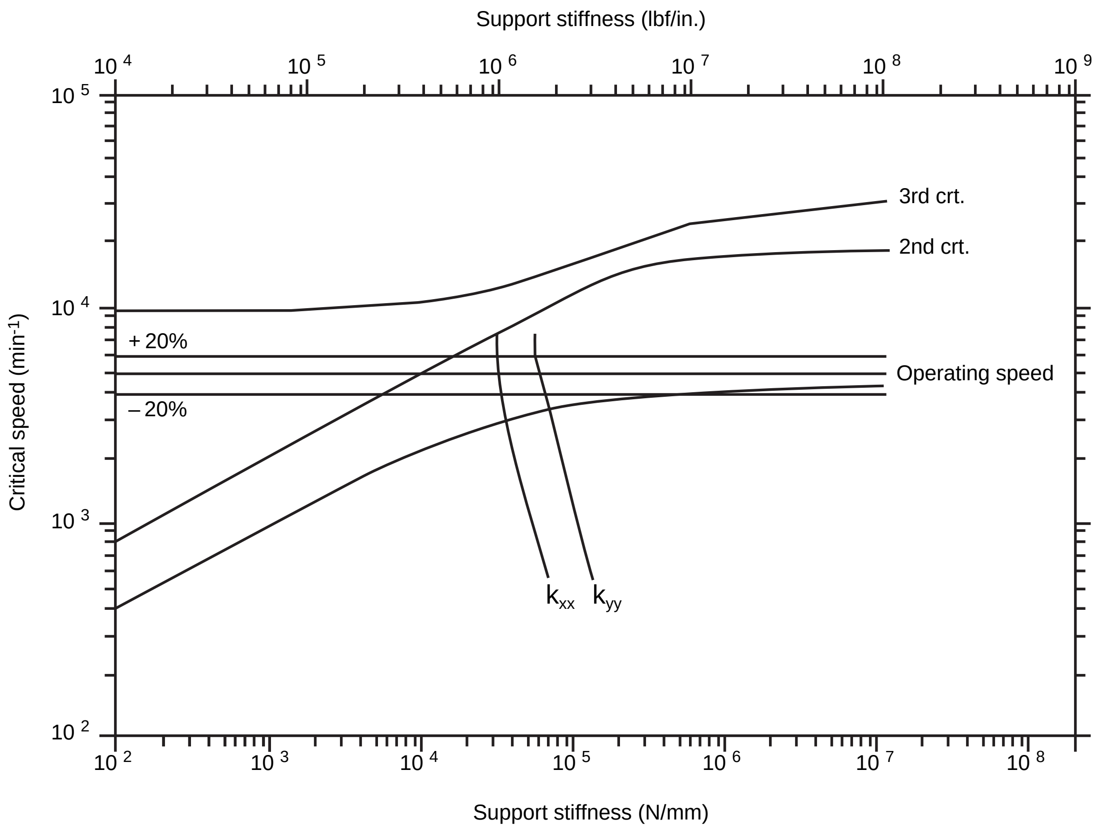


Figure 1-46—Undamped Critical Speed Map for an Eight-Stage 12 MW (16,000 HP) Steam Turbine

Note that the bearing support stiffness and the bearing oil film stiffness may be viewed as two springs in series. Figure 1-47 shows a rotor with support stiffnesses for the pedestals and the foundation. For most turbomachinery, the support stiffness is much greater than the bearing stiffness and, therefore, has a minimal effect on the rotor dynamic response. Some machines such as some industrial gas turbines, however, have a relatively low support stiffness while others, such as machines using antifriction bearings, have a very high bearing stiffness. In either case, the support stiffness now has a greater effect on rotor response and stability. The factor of $K_{support}/K_{bearing} < 3.5$ is important because it represents the stiffness ratio where support flexibility begins to have a significant influence on the system's critical speeds and response characteristics. API sets the value at 3.5 to ensure that support flexibility is accounted for in the most important cases. For cases with a higher value of support stiffness to bearing stiffness, the accuracy of the model may still benefit by accounting for support flexibility.

Extra Bodé plots are required for this case because radial vibration probes can only measure the displacement of the rotor relative to the bearing housing. Absolute vibration takes into account the displacement and bearing housing movement. Figure1-48 shows the differences in vibration amplitude possible for a system with a soft support. For systems like this, API requires that the absolute shaft vibration be determined. This is normally done by mounting an accelerometer on top of the radial vibration probe holder. The housing vibration (and hence the probe holder vibration) is then added vectorially to the radial vibration probe signal to yield the absolute vibration. The ability to distinguish between shaft relative vibration versus absolute vibration is particularly important when very flexible bearing supports are present. Conversely, measurement of bearing housing motion alone can be inadequate in the case of very stiff supports.

1.6.2.2 Phase II—Design Acceptance Criteria (refer to Figure 1-33)

SP 2.8.2.5 The damped unbalanced response analysis shall indicate that the machine in the unbalanced condition described in 2.8.2.4, Item b, will meet the following acceptance criteria (See Figure 1): [2.8.1.6; 2.8.2.4, Item d; 2.8.3.3, Item a]

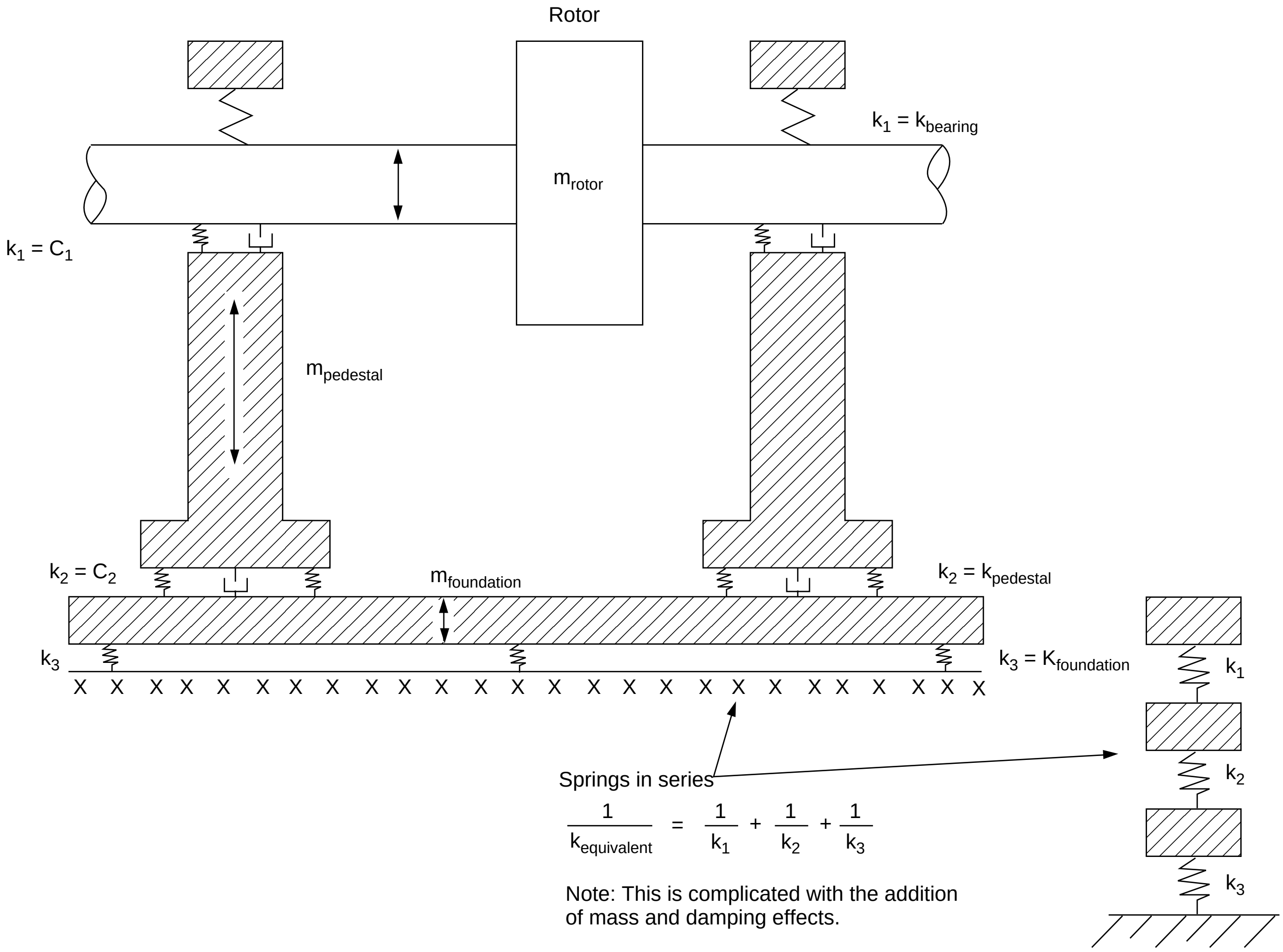


Figure 1-47—Schematic of a Rotor With Flexible Supports and Foundation

a. If the amplification factor is less than 2.5, the response is considered critically damped and no separation margin is required.

b. If the amplification factor is 2.5–3.55, a separation margin of 15 percent above the maximum continuous speed and 5 percent below the minimum operating speed is required.

c. If the amplification factor is greater than 3.55 and the critical response peak is below the minimum operating speed, the required separation margin (a percentage of minimum speed) is equal to the following:

$$SM = 100 - \left[84 + \frac{6}{AF-3}\right] \quad (3)$$

d. If the amplification factor is greater than 3.55 and the critical response peak is above the trip speed, the required separation margin (a percentage of maximum continuous speed) is equal to the following:

$$SM = \left[126 - \frac{6}{AF-3}\right] - 100 \quad (4)$$

In this section, API establishes the first part of the acceptance criteria that a new machine design must meet. This paragraph establishes separation margins for critical speeds (above and below the operating speed range) based on the amplification factor associated with the critical and the location of the critical or criticals of concern. The rules governing critical speed separation margins may be summarized as follows:

a. Increased separation margins are required for criticals with higher amplification factors.
b. Increased separation factors are required for criticals above the operating speed range verses those below the operating speed range.

Variable critical speed separation margins have been established to account for the severity of response of the rotor

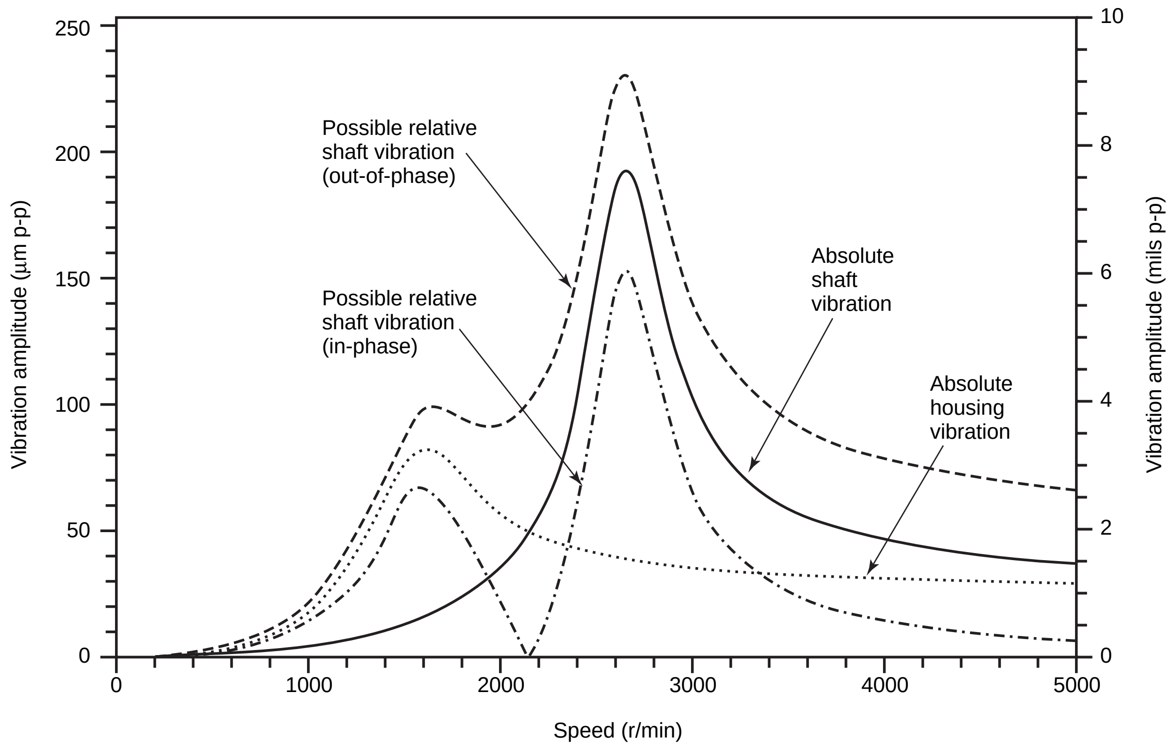


Figure 1-48—Example of Absolute Versus Relative Shaft Vibration For a Flexibly Supported Rotor Bearing System

during operation near the critical of interest. The acceptance criteria are displayed graphically in Figures 1-49 and 1-50 for operation above and below a critical speed, respectively.

At this point, it must again be stressed that these standard paragraphs are superseded by the standards that govern specific machinery. For example, API Standard 613 requires that gear units operate below their first critical speeds with a 20 percent margin (unloaded case). API Standard 541 requires that special purpose induction motors (motors driving unspared critical equipment) operate below the first critical speed with only a 15 percent margin. These examples stress the importance of using the actual specifications established for particular types of machines.

2.8.2.6 The calculated unbalanced peak-to-peak rotor amplitudes (see 2.8.2.4, Item b) at any speed from zero to trip shall not exceed 75 percent of the minimum design diametral running clearances throughout the machine (with the exception of floating-ring seal locations). [2.8.2.7; 2.8.3.2; 2.8.3.3, Item b]

This paragraph outlines the second criteria that a proposed design must meet to be considered acceptable. In order for turbomachinery to provide safe, reliable, and efficient service for extended periods of time, the design must not allow rubs between the rotating and stationary components even when the maximum allowable unbalance is present. For this reason, API requires that the vibration amplitudes not exceed 75 percent of the running clearances. This requirement provides a 25 percent margin before a rub occurs.

Rubs typically occur in turbomachinery at bearing and seal locations. It is important to note that the running clearances are not necessarily the static clearances used during unit assembly because the rotating assembly grows radially (*centrificates*) during operation. Not all components centrificate equally: the OD of an impeller eye may grow by 508 micrometers (20 mils) at MCOS while the radial growth of a bearing journal at the same speed is negligible. Thus, maximum closure of the laby seal clearances may well come at MCOS and not when the unit operates near a critical speed. *Hot* equipment may also experience closure of critical clearances because of differential thermal growths between the rotating and stationary elements.

SP 2.8.2.7 If, after the purchaser and the vendor have agreed that all practical design efforts have been exhausted, the analysis indicates that the separation margins still cannot be met or that a critical response peak falls within the operating speed range, acceptable amplitudes shall be mutually agreed upon by the purchaser and the vendor, subject to the requirements of 2.8.2.6.

Required Separation Margins for Operation Above a Critical Speed	
Amplification factor associated with a critical speed	Required separation margin between the critical speed and minimum operating speed
AF < 2.5	No separation margin required; critically damped response considered
$2.5 \le AF \le 3.55$	Minimum 5% separation margin with minimum operating speed required
3.55 < AF	Minimum S.M. with minimum operating speed defined as follows: Minimum S.M. $= 100 - [84 + \frac{6}{AF - 3}]$

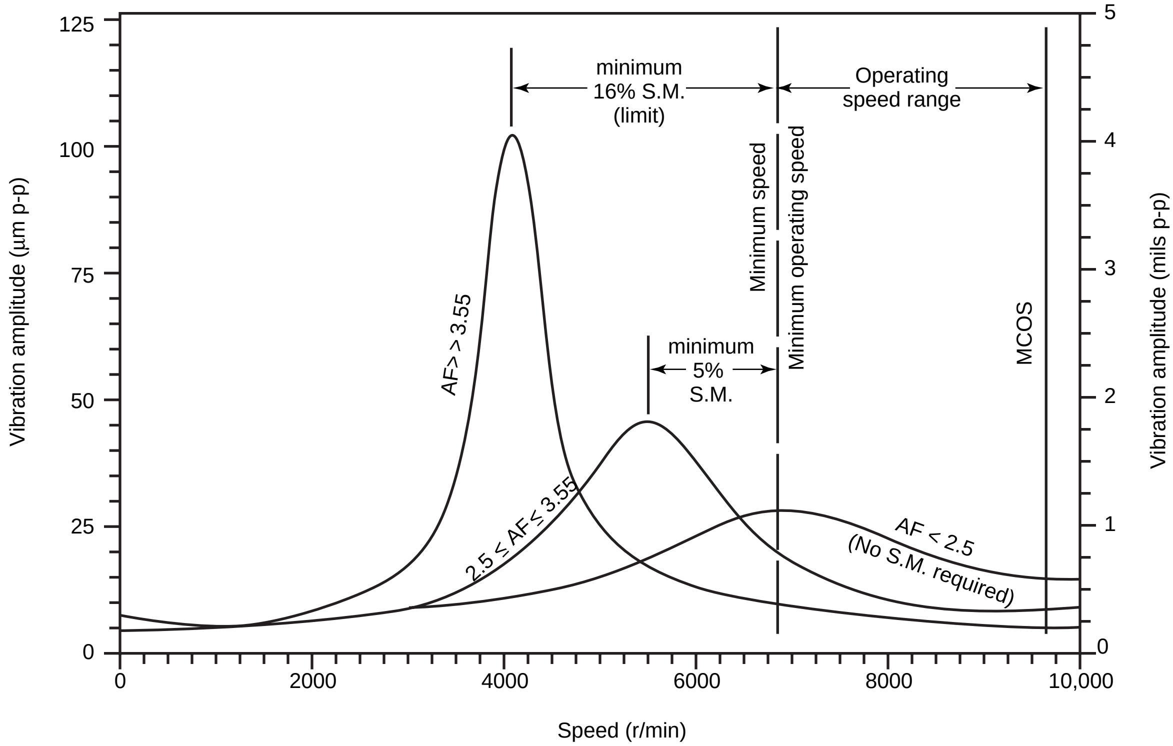


Figure 1-49—API Required Separation Margins for Operation Above a Critical Speed

Required Separation Margins for Operation Above a Critical Speed	
Amplification factor associated with a critical speed	Required separation margin between the critical speed and minimum operating speed
AF < 2.5	No separation margin required; critically damped response considered
2.5 ≤ AF ≤ 3.55	Minimum 5% separation margin with minimum operating speed required
3.55 < AF	Minimum separation margin with maximum operating speed (MCOS) defined as follows: Minimum S.M. = $[126 - \frac{6}{AF - 3}] - 100$

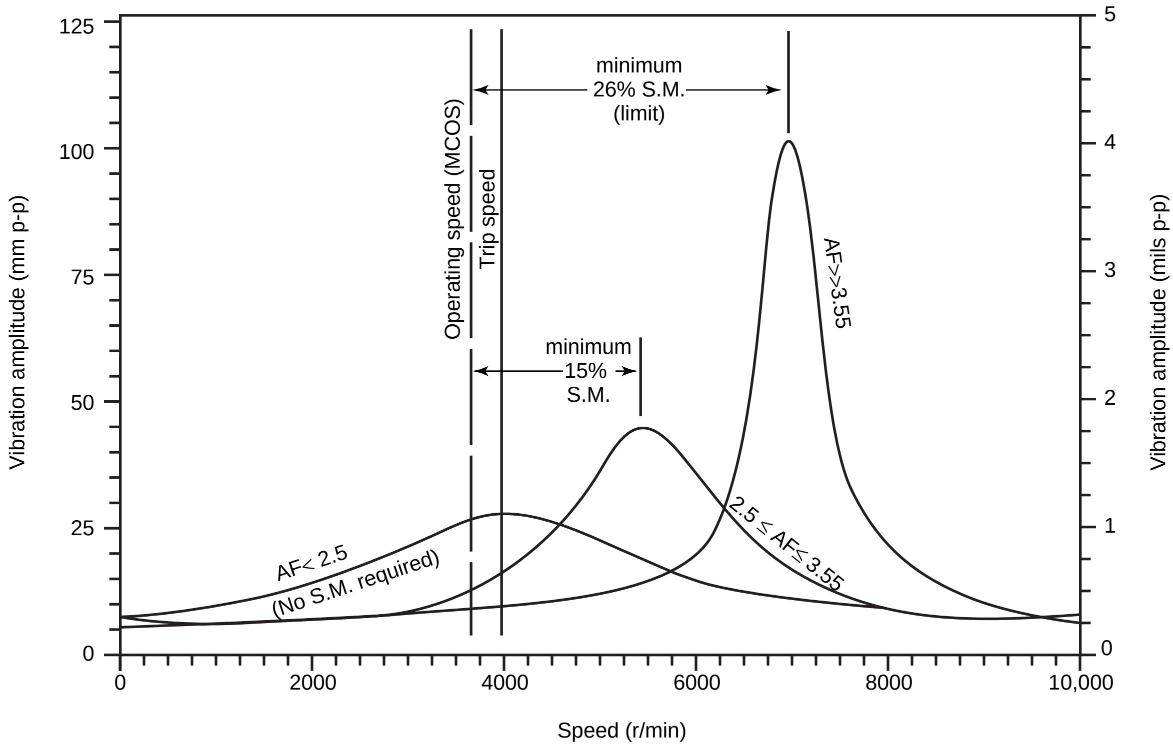


Figure 1-50—API Required Separation Margins for Operation Below a Critical Speed

In rare cases, a purchaser's procurement specifications or the specific demands of an application may produce a design that cannot meet requirements outlined in Standard Paragraphs 2.8.2.5 and 2.8.2.6. For example, a high efficiency steam turbine may possess labyrinth seals that do not possess acceptable running clearances according to 2.8.2.6 at critical speeds or at unit MCOS. In such cases, the manufacturer must demonstrate the following to the satisfaction of the purchaser:

a. The unit cannot be re-designed to achieve compliance without significantly compromising commercial terms, performance guarantees, or other design criteria.
b. The unit is capable of safe and reliable operation under all anticipated operating conditions.

This paragraph emphasizes the dominant criteria for safe unit operation: the unit shall operate without contact between the stationary and rotating elements plus a margin of safety.

This completes Phase I and Phase II of API's rotor dynamics acceptance program. At this point, the proposed machine design has been modeled, the lateral rotor dynamics characteristics of the unit have been calculated, and the unit's design has been evaluated and accepted based on the following criteria:

a. Adequate critical speed separation margins and
b. Acceptable minimum running clearances.

Or the manufacturer has demonstrated the following to the satisfaction of the purchaser:

a. The proposed design cannot be improved.
b. The assembled unit is capable of safe and reliable operation.

1.6.2.3 Phase III—Shop Model Verification Testing and Unit Acceptance (see Figure 1-33)

SP 2.8.3 SHOP VERIFICATION OF UNBALANCED RESPONSE ANALYSIS [2.8.2.4, Item d; 2.8.5.5; 4.3.3.3.3; 4.3.3.3.5]

The principal concern of Phases I and II (see 1.5.2.1 and 1.5.2.2) of the API rotor dynamics acceptance program is for the manufacturer to generate equipment designs that possess desirable lateral dynamic characteristics such as critical speed separation margins. The main goal of Phase III is simply to ensure that the computer models generated prior to manufacture are representative of the actual equipment as it operates in the field. Despite the presence of unit vibration acceptance criteria in Standard Paragraph 2.8.3.4, Subsection 2.8.3, does not directly address vibration and balancing acceptance criteria; these topics are discussed in 2.8.5 of the API *Standard Paragraphs*.

While the concern for model validation may seem strange, it must be noted that the mechanical condition of the newly assembled unit can only degrade with time. For example, a well-balanced centrifugal compressor may operate in close proximity to an amplified critical speed without excessive vibration, but the unbalanced condition of the unit and attendant vibrations will degrade during operation if the process gas coats the impellers or blades with solid deposits (known as *coking*), or if the process gas is erosive or corrosive. Thus, while a unit may successfully pass a mechanical run test and operate in the field for a period of time, unless the rotor system design possesses acceptable lateral characteristics, prolonged safe and reliable operation cannot be guaranteed.

The model validation process is described in detail in Figure 1-34. If the shop test vibration measurements correlate with calculated results for shop test conditions within the tolerances specified in Standard Paragraph 2.8.3.2.2 and the unit conforms to the acceptance criteria in Standard Paragraph 2.8.3.3, then all analysis and shop testing work are concluded, the purchaser is issued appropriate analysis reports and required quality documentation, and the unit is shipped. If the shop test vibration measurements do not correlate with results calculated for shop test conditions, then the computer model must be adjusted until agreement within the specified tolerances is attained. If the model adjustments affect the computer model of the unit operating in the field, then the complete lateral analysis may have to be performed again as well. According to Standard Paragraph 2.8.3.4, additional shop testing is required only if the vibration measurements indicate that a rotor critical speed or a bearing support resonance interferes with the unit's operating speed range.

SP 2.8.3.1 **A demonstration of rotor response at future unbalanced conditions is necessary because a well-balanced rotor may not be representative of future operating conditions (see 2.8.2.4, Item d). This test shall be performed as part of the mechanical running test (see 4.3.3), and the results shall be used to verify the analytical model. Unless otherwise specified, the verification test of the rotor unbalance shall be performed only on the first rotor (normally the spare rotor, if two rotors are purchased).**

Shop verification testing is performed after completion of the four-hour mechanical run test (including the overspeed tests, if required). The mechanical run test is discussed in 2.8.5.

Normally, the main and the spare rotors are virtually identical; therefore, only one of the rotors must be subjected to the unbalanced run testing.

SP 2.8.3.1 (continued) **The actual response of the rotor on the test stand to the same unbalance weight as was used to develop the Bodé plots specified in 2.8.2.4 shall be the criterion for determining the validity of the damped unbalanced response analysis. To accomplish this, the fol-**

lowing procedure shall be followed:

The general purpose of the shop verification test is to determine the lateral response characteristics (critical speeds and associated amplification factors) of the assembled unit up to the unit's maximum operating speed. This procedure requires placing pre-determined unbalances on the rotating assembly in order to raise the vibration amplitude at the operating speed nearest the critical of concern to the API vibration limit specified in Standard Paragraph 2.8.2.4.

Great care must be exercised when placing unbalance on a rotor because operating a rotor with excessive unbalance might permanently bow the shaft or otherwise damage the unit. For this reason, the procedure in this Standard Paragraph 2.8.3.1 has been developed to safely determine the unbalance needed to raise rotor vibrations to the API vibration limit. Also, the procedure outlined in this paragraph is based on the method of influence coefficients developed for rotor balancing. This method requires placing a trial unbalance on the rotor and measuring, at a given speed, the change in rotor response from the balanced condition. Once the influence of the added unbalance has been calculated, the unbalance needed to increase vibration levels to the API limit can be determined.

SP 2.8.3.1, a. During the mechanical running test (see 4.3.3), the amplitudes and phase angle of the indicated vibration at the speed nearest the critical or criticals of concern shall be determined.

All rotating elements balanced to within API specifications (see 2.8.5) contain a finite amount of residual unbalance that causes vibration during operation. The effect of this residual unbalance must be considered when adding the test unbalance to the rotor in order to minimize the possibility of damaging the unit during shop verification testing. For this reason, the response of the rotor at operating speeds nearest the criticals of concern must be recorded.

The residual unbalance in the balanced rotor generates a rotating force vector that causes the rotor to bow out from the center of rotation and orbit in a closed path at the frequency of shaft rotation (synchronous frequency). Non-contacting displacement probes measure the size of the gap between the probe and the rotating element. An orbiting shaft will cause the probe gap to increase and decrease over the course of one shaft revolution. Sample probe signals for the filtered synchronous waveforms and the resulting orbit are displayed in Figure 1-51. Given the time-varying size of the probe gaps, the amplitude and phase of the maximum shaft displacement are electronically determined for each probe. Note that the phase can only be measured relative to an arbitrary reference point on the shaft, typically defined by a notch in the shaft, such as a keyway.

As previously noted in the discussion of Standard Paragraph 2.8.2.4, when the rotor operating speed range is less than the first critical speed there is only one critical of concern. The rotor's vibration amplitude and phase are, therefore, recorded at the operating speed closest to this critical. If the rotor operating speed range is located between two critical speeds, then all critical speeds below the minimum operating speed plus the critical immediately above the maximum operating speed are considered criticals of concern. For this case, the rotor's vibration amplitude and phase are recorded at both the minimum and maximum operating speeds.

SP 2.8.3.1, b. A trial weight, not more than one-half the amount calculated in 2.8.2.4, Item b, shall be added to the rotor at the location specified in 2.8.2.4, Item d; 90 degrees away from the phase of the indicated vibration at the speed or speeds closest to the critical or criticals of concern.

A trial weight smaller in size than the calculated test weight is applied to the rotating assembly to determine the response sensitivity of the criticals of concern. A trial weight is initially used instead of the calculated test unbalance weight in case the response sensitivity of the unit to the applied unbalance is substantially greater than the sensitivity predicted during the computer analysis. Additionally, the trial weight should not to be applied in-phase (0 degrees relative to the residual unbalance) or out-of-phase (180 degrees relative to the residual unbalance) with the existing residual unbalance for the following reasons:

a. Placing the trial weight in-phase with the existing residual unbalance might potentially damage the rotor if the criticals of concern are sensitive to applied unbalance.
b. Placing the trial weight out-of-phase with the existing residual unbalance might significantly decrease the response of the rotor and increase the uncertainty in the vibration measurements. For example, a minor shaft bow or preset might substantially affect test measurements if the net unbalance (residual plus trial unbalances) is small.

SP 2.8.3.1, c. The machine shall then be brought up to the operating speed nearest the critical of concern, and the indicated vibration amplitudes and phase shall be measured. The results of this test and the corresponding indicated vibration from Item a above shall be vectorially added to determine the magnitude and phase location of the final test weight required to produce the required test vibration amplitudes.

Once the trial weight is applied, the machine is run at the operating speed nearest the critical of concern and the resulting filtered synchronous vibration is recorded. The amplitude and phase of the required test weight can be calculated using the procedure outlined in the following.

Since this procedure is so important, the procedure is graphically illustrated in Figure 1-52. This figure contains sample vector diagrams and mathematical formulae that can

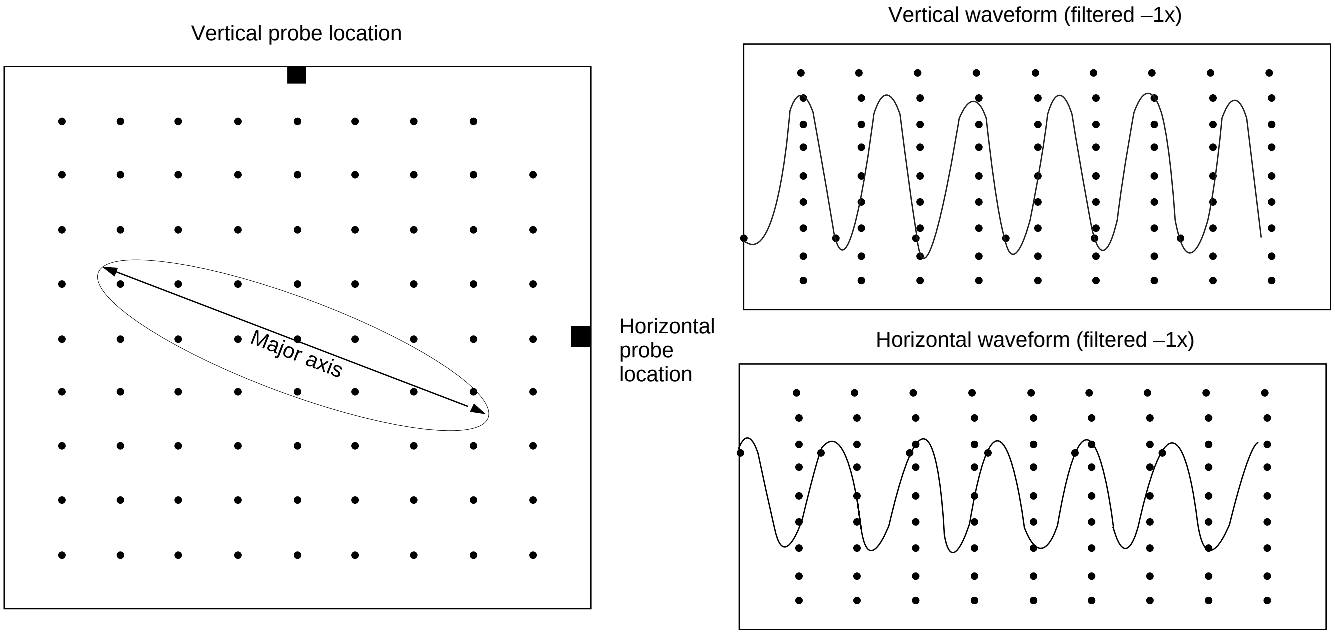


Figure 1-51—Determination of Major Axis Amplitude From a Lissajous Pattern (Orbit) on an Oscilloscope

be used to calculate the test unbalance.

SP 2.8.3.1, d. The final test weight described in Item c above shall be added to the rotor, and the machine shall be brought up to the operating speed nearest the critical of concern. When more than one critical of concern exists, additional test runs shall be performed for each, using the highest speed for the initial test run.

If there is more than one critical of concern, the criticals of concern should be unbalanced in reverse order. The reason for this can be understood by examining the relative level of unbalance necessary to fulfill test requirements. Consider the case of a bending mode above trip speed where the unbalance weight would be a function of overhung (O/H) mass only:

Assume:

O/H mass	=	355.9 Newtons (80 pound-feet).
Trip speed	=	14,300 revolutions per minute.
MCOS	=	13,000 revolutions per minute.
Minimum speed	=	7500 revolutions per minute.
First mode	=	6000 revolutions per minute.
Total mass	=	4448.2 Newtons (1000 pound-feet).

For this case, the required unbalance for the second mode is:

In SI units,

$$L_v = \sqrt{\frac{12,000}{N}}$$

$$\text{Second mode unbalance} = 2 \times \left[\frac{4 \times 80}{13,000}\right] = 0.05 \text{ ounce-inch}$$

For this same rotor, the required minimum first mode test unbalance is:

In SI units,

$$L_v = \sqrt{\frac{12,000}{N}}$$

$$\text{First mode unbalance} = 2 \times \left[\frac{4 \times 1000}{7500}\right] = 1.07 \text{ ounce-inches}$$

The first mode unbalance is over 21 times the second mode unbalance. A cautionary approach requires that the lighter unbalance be placed on the rotor first.

SP 2.8.3.1 Note: It is recognized that the dynamic response of the machine on the test stand will be a function of the agreed-upon test conditions and that unless the test-stand results are obtained at the conditions of pressure, temperature, speed, and load expected in the field, they may not be the same as the results expected in the field.

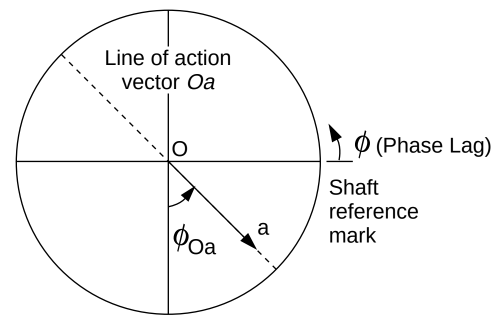
This note alludes to the proper interpretation of machine response on open-versus-closed loop testing, testing on job seals versus test seals at reduced pressure, and so on. These differences can be extremely important: in some cases, the dynamic performance of the unit during the shop test will be dramatically different from the unit operating in the field.

The concerns expressed in this note are particularly aimed at high pressure centrifugal compressors tested at reduced

The unbalance calculation procedure is outlined in detail below. Including sample vector diagrams and mathematical formulas. The unbalance and response vector diagrams have been separated for clarity.

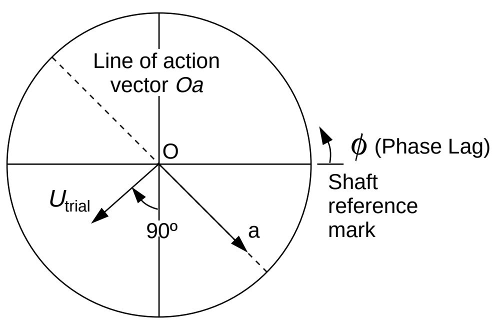
Step 1. Measure vibration *Oa* at the operating speed nearest the critical speed of concern. Vibration vector *Oa* is generated by the residual unbalance in the balanced rotor.

RESPONSE VECTOR DIAGRAM

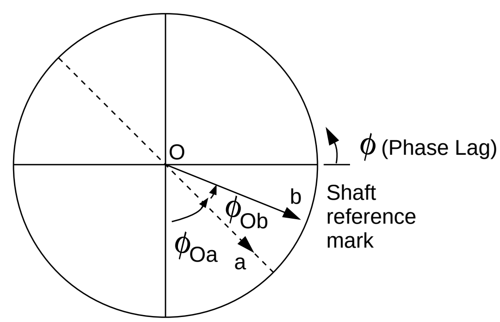


Step 2. Apply the trial unbalance, U_{trial}, and measure the resulting vibration vector, *Ob*, at the operating speed nearest the critical speed of concern. Vibration vector *Ob* is generated by the residual plus the trial unbalances.

UNBALANCE VECTOR DIAGRAM

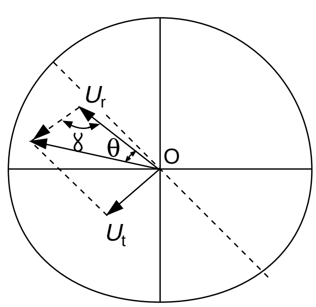


RESPONSE VECTOR DIAGRAM



Step 3. Generate response and unbalance vector triangles and then calculate the residual unbalance vector, $U_{residual}$. Note that the two triangles displayed in these diagrams are assumed to be geometrically similar.

UNBALANCE VECTOR DIAGRAM



RESPONSE VECTOR DIAGRAM

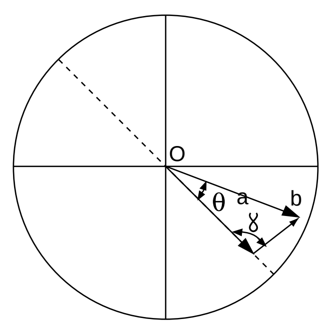


Figure 1-52—Calculating the Test Unbalance

Step 3. (Cont'd)

The magnitude of the residual unbalance is the following:

$$U_{residual} = U_{trial} \mid Oa/ab \mid$$

ab is determined from the following:

$$\mid ab \mid^2 = \mid Oa \mid^2 + \mid Ob \mid^2 - 2 \mid ab \mid \cos\theta$$

The direction of the residual unbalance vector is identified in Figure 51. This angle can be calculated using the law of sines:

$$(\sin \gamma) / \mid Ob \mid = (\sin \theta) / \mid ab \mid$$

On the rotating element γ is measured in the direction of phase lag (opposite the direction of rotation).

Step 4. Remove the trial weight and minimize the residual unbalance by placing a correction unbalance equal to $U_{residual}$ that is 180° out-of-phase with the calculated residual unbalance vector, $U_{residual}$.

Step 5. Place C x $U_{residual}$ in the direction of $U_{residual}$ to get a vibration level equal to C x *Oa*, to raise the vibration level to the required test level.

Step 6. Continue the verification test. At this point, the rotor should be sufficiently unbalanced to raise the measured vibrations at the operating speed nearest the critical of concern to the API vibration limit.

Figure 1-52—Calculating the Test Unbalance (Continued)

pressure with special test seals. Figure 1-26 displays the differences in the unbalance response of a unit for various suction pressures. As the seal oil supply is generally referenced to the suction pressure of the unit, these curves also display the potential effect of installing low pressure test seals on the response of a unit during the mechanical run. When the unit is in service, the unit's suction pressure is much higher than during the run test, and the rotor's response during test can be much increased due to the diminished damping provided by the oil seals. For this reason, the rotor may be damaged during test if operated near a critical speed for a prolonged period of time.

Even if the job oil seals are used during the test, the full destabilizing influence of the seals are not experienced by the unit during the test because of the reduced sealing pressures. Thus, the unit may operate satisfactorily during the test and run with violent subsynchronous (unstable) vibrations during full pressure field operation.

SP 2.8.3.2 The parameters to be measured during the test shall be speed and shaft synchronous (1×) vibration amplitudes with corresponding phase. The vibration amplitudes and phase from each pair of x-y vibration probes shall be vectorially summed at each response peak to determine the maximum amplitude of vibration. The major-axis amplitudes of each response peak shall not exceed the limits specified in 2.8.2.6 (More than one application of the unbalance weight and test run may be required to satisfy these criteria).

The gain of the recording instruments used shall be predetermined and preset before the test so that the highest response peak is within 60–100 percent of the recorder's full scale on the test-unit coast-down (deceleration; see 2.8.3.4). The major-axis amplitudes at the operating speed nearest the critical or criticals of concern shall not exceed the values predicted in accordance with 2.8.2.4, Item d, before coast-down through the critical of concern.

This paragraph defines the parameters to be measured and plotted during the shop verification test. Bodé plots for each probe shall be generated during the verification test with the maximum amplitude of response greater than 60 percent of the plot's full scale. This requirement ensures that critical speeds can be accurately identified. A representative Bodé plot is displayed in Figure 1-53.

This paragraph also requires that the rotor be capable of safe operation when operating in the unbalanced condition defined in Standard Paragraph 2.8.2.4. Specifically, when the rotor is unbalanced at the operating speed nearest the critical of concern, the unit must be capable of traversing the critical speeds without exceeding 75 percent of the running clearances. As the rotor displacements are measured only at probe locations, the rotor displacements in the bundle must be calculated using the mode shapes calculated for the unbalances used during the test. As previously noted, all critical speeds below the operating speed range of the rotor plus the critical immediately above the operating speed range are the criticals of concern.

SP 2.8.3.2.1 Vectorial addition of slow-roll (300–600 revolutions per minute) electrical and mechanical runout is required to determine the actual vibration amplitudes and phase during the verification tests. Vectorial addition

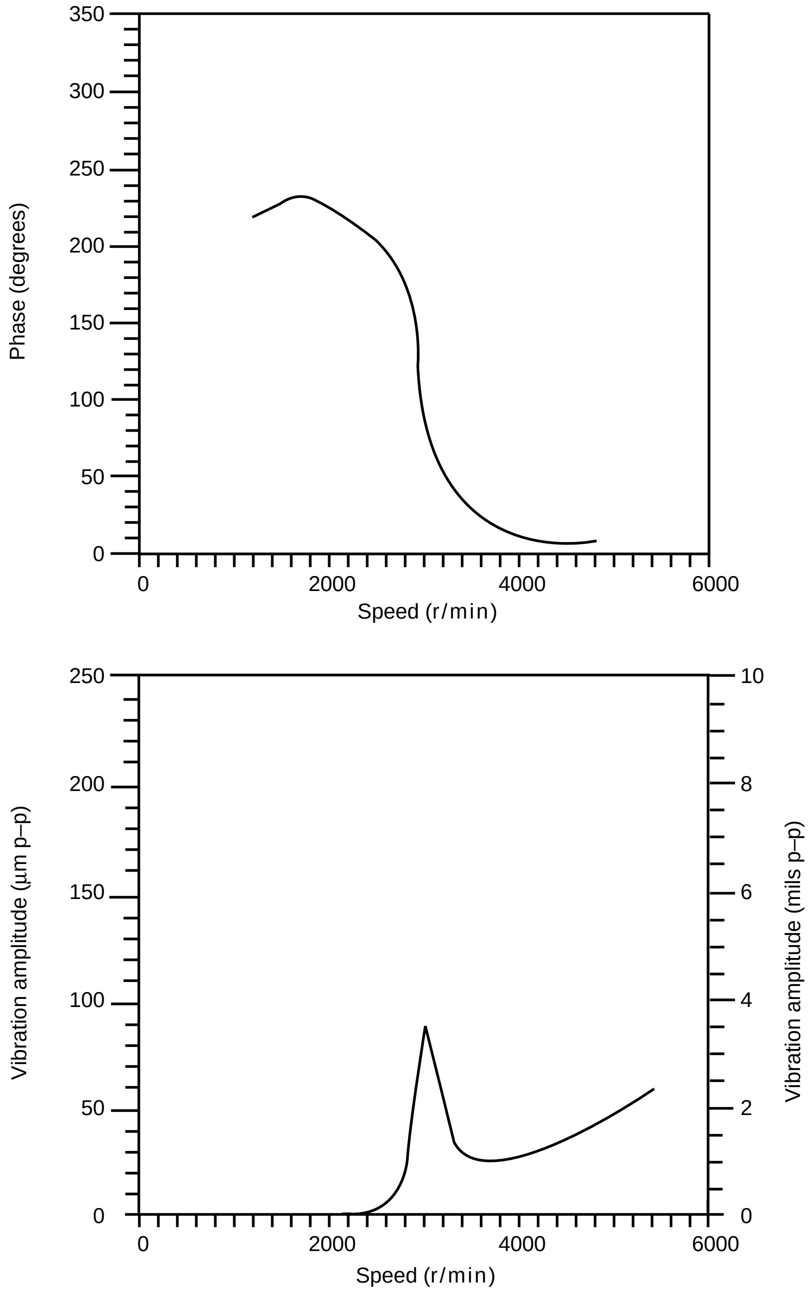


Figure 1-53—Bode Plot For First Mode Test Unbalance

of the bearing-housing motion is required for machines that have flexible rotor supports (see 2.8.2.4, Item f).

The effect of electrical and mechanical runouts must be eliminated by electronically subtracting the measured slow-roll vibration waveform from the measured vibrations at operating speed. This elimination is required to ensure that the measured vibration levels accurately reflect the rotor's response to unbalance. In addition, for machines with flexible supports, the bearing housing vibration must likewise be vectorially added to ensure that bearing housing motion is not biasing the measured vibration levels. Figure 1-48 displays the potential dynamic influence of support effects.

Note 1: The phase on each vibration signal, *x* or *y*, is the angular measure, in degrees, of the phase difference (lag) between a phase reference signal (from a phase transducer sensing a once-per-revolution mark on the rotor, as described in API Standard 670) and the next positive peak, in time, of the synchronous (1×) vibrational signal. (A phase change will occur through a critical or if a change in a rotor's balance condition occurs because of shifting or looseness in the assembly.)
Note 2: The major-axis amplitude is properly determined from a lissajous (orbit) display on an oscilloscope or equivalent instrument. When the phase angle between the *x* and *y* signals is not 90 degrees, the major-axis amplitude can be approximated by $(x^2 + y^2)^{0.5}$. When the phase angle between the *x* and *y* signals is 90 degrees, the major-axis amplitude value is the greater of the two vibration signals.

Figure 1-54 presents an elliptical orbit of shaft vibration showing the major and minor axis. Figure 1-54 shows actual vibration data for a running machine, including a lissajous orbit display with the major axis indicated. This major axis value is greater than the vibration levels indicated by either the x or y probes, and this major axis value represents the true peak vibration levels the machine is experiencing.

SP 2.8.3.2.2 The results of the verification test shall be compared with those from the original analytical model. The vendor shall correct the model if it fails to meet any of the following criteria:

a. The actual critical speeds shall not deviate from the predicted speeds by more than ± 5 percent.
b. The predicted amplification factors shall not deviate from the actual test-stand values by more than ± 20 percent.
c. The actual response peak amplitudes, including those that are critically damped, shall be within ± 50 percent of the predicted amplitudes.

At this point, the API *Standard Paragraphs* require that the unbalance response predictions from the computer model

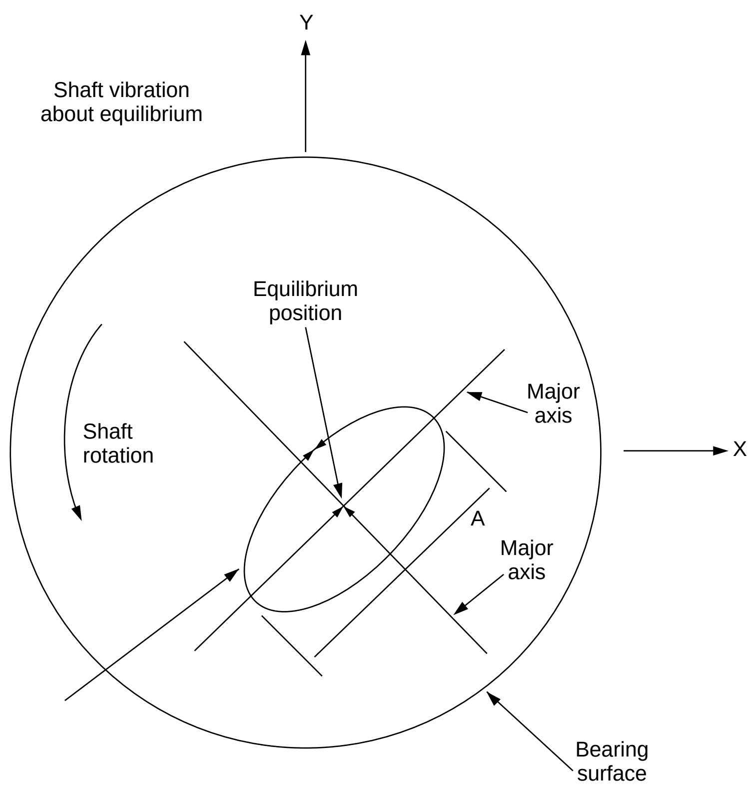


Figure 1-54—Elliptical Orbit of Shaft Vibration Showing Major and Minor Axes of Lissajous Pattern

and analysis be compared to the unbalance shop test results from the actual machine. The analytical predictions must correlate with the shop test results within the specified ranges, or else the model is rejected and must be corrected. An accurate model is important because it permits accurate evaluation of the unit for future operating or unbalance conditions. The revised response analysis is evaluated for appropriate separation margins and running clearances; and if requirements are not met, then additional testing of the unit may be required. If the machine fails the additional tests, then the model can be used to determine and evaluate appropriate redesign modifications.

SP 2.8.3.3 **Additional testing is required if, from the test data described above or from the damped, corrected unbalanced response analysis (see 2.8.3.2.2), it appears that either of the following conditions exists:**

a. Any critical response will fail to meet the separation margin requirements (see 2.8.2.5) or will fall within the operating speed range.
b. The requirements of 2.8.2.6 have not been met.

This paragraph requires additional testing if either the test stand data or the revised response analysis indicates that separation margin requirements will not be met. In addition, further testing is required for flexible support systems to establish Bodé plots that compare absolute shaft motion with shaft motion relative to the bearing housing.

SP 2.8.3.4 **Rotors requiring additional testing per 2.8.3.3 shall be tested as follows: Unbalance weights shall be placed as described in 2.8.2.4, Item b; this may require disassembly of the machine for placement of the unbalance weights. Unbalance magnitudes shall be achieved by adjusting the indicated unbalance that exists in the rotor from the initial run to raise the displacement of the rotor at the probe locations to the vibration limit defined by Equation 1 (see 2.8.2.4, Item b) at the maximum continuous speed; however, the unbalance used shall be no less than twice the unbalance limit specified in 2.8.5.2. The measurements from this test, taken in accordance with 2.8.3.2, shall meet the following criteria: [2.8.3.2]**

a. At no speed outside the operating speed range, including the separation margins, shall the shaft deflections exceed 90 percent of the minimum design running clearances.
b. At no speed within the operating speed range, including the separation margins, shall the shaft deflections exceed 55 percent of the minimum design running clearances or 150 percent of the allowable vibration limit at the probes (see 2.8.2.4, Item b).

The internal deflection limits specified in Items a and b above shall be based on the calculated displacement ratios between the probe locations and the areas of concern identified in 2.8.2.4, Item c. Actual internal displacements for these tests shall be calculated by multiplying these ratios by the peak readings from the probes. Acceptance will be based on these calculated displacements or on inspection of the seals if the machine is opened. Damage to any portion of the machine as a result of this testing shall constitute failure of the test. Minor internal seal rubs that do not cause clearance changes outside the vendor's new-part tolerance do not constitute damage.

The final tests outlined in this paragraph are intended to ensure that a machine which does not meet the established criteria of Standard Paragraph 2.8.3.3 will be capable of successful operation. This testing allows the manufacturer a final opportunity to prove that the unit is capable of safe and reliable operation. If the unit cannot operate according to the refined criteria presented in this paragraph, then the unit must undergo corrective modifications and be re-tested. These criteria are established to ensure that the machine will operate without contact between stationary and rotating components, plus a small margin of safety, despite response characteristics which do not meet the criteria of 2.8.3.3. Note that the mode shapes calculated for the unbalances applied during this test are used to determine the closure of critical clearances given displacement probe vibration measurements.

This paragraph specifies that the machine may have to be opened to place the unbalance weights. Until now, all attempts have been made to avoid opening the machine. Disassembly of the machine is considered a last resort reserved only for those machines which do not meet with the standard acceptance criteria established in Standard Paragraphs 2.8.3.2 and 2.8.3.3.

APPENDIX 1A—API STANDARD PARAGRAPHS, SECTIONS 2.8.1–2.8.3 ON CRITICAL SPEEDS, LATERAL ANALYSIS, AND SHOP VERIFICATION TESTING; AND SECTION 4.3.3 ON MECHANICAL RUNNING TEST

The following are unannotated excerpts from API *Standard Paragraphs*, 2.8.1–2.8.3, on critical speeds, lateral analysis, and shop verification testing; and 4.3.3 on mechanical running testing:

2.8.1 CRITICAL SPEEDS

2.8.1.1 When the frequency of a periodic forcing phenomenon (exciting frequency) applied to a rotor-bearing support system coincides with a natural frequency of that system, the system may be in a state of resonance.

2.8.1.2 A rotor-bearing support system in resonance will have its normal vibration displacement amplified. The magnitude of amplification and the rate of phase-angle change are related to the amount of damping in the system and the mode shape taken by the rotor.

Note: The mode shapes are commonly referred to as the first rigid (translatory or bouncing) mode, the second rigid (conical or rocking) mode, and the (first, second, third, ..., *n*th) bending mode.

2.8.1.3 When the rotor amplification factor (see Figure 1), as measured at the shaft radial vibration probes, is greater than or equal to 2.5, the corresponding frequency is called a *critical speed,* and the corresponding shaft rotational frequency is also called a critical speed. For the purposes of this standard, a critically damped system is one in which the amplification factor is less than 2.5.

2.8.1.4 Critical speeds and their associated amplification factors shall be determined analytically by means of a damped unbalanced rotor response analysis and shall be confirmed during the running test and any specified optional tests.

2.8.1.5 An exciting frequency may be less than, equal to, or greater than the rotational speed of the rotor. Potential exciting frequencies that are to be considered in the design of rotor-bearing systems shall include but are not limited to the following sources:

a. Unbalance in the rotor system.
b. Oil-film instabilities (whirl).
c. Internal rubs.
d. Blade, vane, nozzle, and diffuser passing frequencies.
e. Gear-tooth meshing and side bands.
f. Coupling misalignment.
g. Loose rotor-system components.
h. Hysteretic and friction whirl.
i. Boundary-layer flow separation.
j. Acoustics and aerodynamic cross-coupling forces.
k. Asynchronous whirl.
l. Ball and race frequencies of anti-friction bearings.

2.8.1.6 Resonances of structural support systems may adversely affect the rotor vibration amplitude. Therefore, resonances of structural support systems that are within the vendor's scope of supply and that affect the rotor vibration amplitude shall not occur within the specified operating speed range or the specified separation margins (see 2.8.2.5) unless the resonances are critically damped.

2.8.1.7 The vendor who is specified to have unit responsibility shall determine that the drive-train (turbine, gear, motor, and the like) critical speeds (rotor lateral, system torsional, blading modes, and the like) will not excite any critical speed of the machinery being supplied and that the entire train is suitable for the specified operating speed range, including any starting-speed detent (hold-point) requirements of the train. A list of all undesirable speeds from zero to trip shall be submitted to the purchaser for his review and included in the instruction manual for his guidance (see Appendix C, Item 42).

2.8.2 LATERAL ANALYSIS [4.3.3.3.3]

2.8.2.1 The vendor shall provide a damped unbalanced response analysis for each machine to assure acceptable amplitudes of vibration at any speed from zero to trip.

2.8.2.2 The damped unbalanced response analysis shall include but shall not be limited to the following considerations:

a. Support (base, frame, and bearing-housing) stiffness, mass, and damping characteristics, including effects of rotational speed variation. The vendor shall state the assumed support system values and the basis for these values (for example, tests of identical rotor support systems, assumed values).
b. Bearing lubricant-film stiffness and damping changes due to speed, load, preload, oil temperatures, accumulated assembly tolerances, and maximum-to-minimum clearances.
c. Rotational speed, including the various starting-speed detents, operating speed and load ranges (including agreed upon test conditions if different from those specified), trip speed, and coast-down conditions.
d. Rotor masses, including the mass moment of coupling halves, stiffness, and damping effects (for example, accumulated fit tolerances, fluid stiffening and damping, and frame and casing effects).
e. Asymmetrical loading (for example, partial arc admission, gear forces, side streams, and eccentric clearances).
f. The influence, over the operating range, of the calculated values for hydrodynamic stiffness and damping generated by the casing end seals.

For machines equipped with antifriction bearings, the vendor shall state the bearing stiffness and damping values used for the analysis and either state the basis for these values or the assumptions made in calculating the values.

• **2.8.2.3** When specified, the effects of other equipment in the train shall be included in the damped unbalanced response analysis (that is, a train lateral analysis shall be performed).

Note: This analysis should be considered for machinery trains with coupling spacers greater than 1 meter (36 inches), rigid couplings, or both.

2.8.2.4 As a minimum, the damped unbalanced response analysis shall include the following:

a. A plot and identification of the mode shape at each resonance speed (critically damped or not) from zero to trip, as well as the next mode occurring above the trip speed.

b. Frequency, phase, and response amplitude data (Bodé plots) at the vibration probe locations through the range of each critical speed, using the following arrangement of unbalance for the particular mode. This unbalance shall be sufficient to raise the displacement of the rotor at the probe locations to the vibration limit defined by the following equation:

In SI units,

$$L_v = 25.4\sqrt{\frac{12,000}{N}} \tag{1}$$

In US Customary units,

$$L_v = \sqrt{\frac{12,000}{N}}$$

Where:

L_v = vibration limit (amplitude of unfiltered vibration), in micrometers (mils) peak-to-peak.
N = operating speed nearest the critical of concern, in revolutions per minute.

This unbalance shall be no less than two times the unbalance defined by the following equation:

In SI units,

$$U = 6350W/N \tag{2}$$

In US Customary units:

$$U = 4W/N$$

Where:

U = input unbalance from the rotor dynamic response analysis, in gram-millimeters (ounce-inches).
W = journal static weight load, in kilograms (pounds), or for bending modes where the maximum deflection occurs at the shaft ends, the overhung weight load (that is, the weight outboard of the bearing), in kilograms (pounds).
N = operating speed nearest the critical of concern, in revolutions per minute.

The unbalance weight or weights shall be placed at the locations that have been analytically determined to affect the particular mode most adversely. For translatory modes, the unbalance shall be based on both journal static weights and shall be applied at the locations of maximum displacement. For conical modes, each unbalance shall be based on the journal weight and shall be applied at the location of maximum displacement of the mode nearest the journal used for the unbalance calculation, 180 degrees out of phase. Figure 2 shows the typical mode shapes and indicates the location and definition of U for each of the shapes. [2.8.2.4, Item d; 2.8.2.5; 2.8.2.6; 2.8.3.1, Item b; 2.8.3.4, Item b]

c. Modal diagrams for each response in Item b. above, indicating the phase and major-axis amplitude at each coupling engagement plane, the centerlines of the bearings, the locations of the vibration probes, and each seal area throughout the machine. The minimum design diametral running clearance of the seals shall also be indicated. [2.8.3.4]

d. A verification test of the rotor unbalance to establish the validity of the analytical model. A verification test of the rotor unbalance is required at the completion of the mechanical running test. Therefore, additional plots based on the actual unbalance to be used during this test shall be provided as follows: For machines that meet the requirements of 2.8.2.4, Item b, and 2.8.2.5, additional Bodé plots, as specified in 2.8.2.4, Item b, shall be provided. The location of the test unbalance shall be determined by the vendor. The amount of unbalance shall be sufficient to raise the vibration levels, as measured at the vibration probes, to those specified in 2.8.2.4 , Item b. In all cases, the unbalance plots shall include the effects of any test-stand conditions (including the effects of test seals) that may be used during the verification test of the rotor unbalance (see 2.8.3). [2.8.3.1; 2.8.3.2]

e. Unless otherwise specified, a stiffness map of the undamped rotor response from which the damped unbalanced response analysis specified in Item c above was derived. This plot shall show frequency versus support system stiffness, with the calculated support system stiffness curves superimposed.

f. For machines whose bearing support stiffness values are less than or equal to 3.5 times the bearing stiffness values, the calculated frequency-dependent support stiffness and damping values (impedances) or the values derived from modal testing. The results of the damped unbalanced response analysis shall include Bodé plots that compare absolute shaft motion with shaft motion relative to the bearing housing. [2.8.3.1]

2.8.2.5 The damped unbalanced response analysis shall indicate that the machine in the unbalanced condition described in 2.8.2.4, Item b, will meet the following acceptance criteria (see Figure 1): [2.8.1.6; 2.8.2.4, Item d; 2.8.3.3, Item a]

a. If the amplification factor is less than 2.5, the response is considered critically damped, and no separation margin is required.
b. If the amplification factor is 2.5–3.55, a separation margin of 15 percent above the maximum continuous speed and 5 percent below the minimum operating speed is required.
c. If the amplification factor is greater than 3.55 and the critical response peak is below the minimum operating speed, the required separation margin (a percentage of minimum speed) is equal to the following:

$$SM = 100 - \left[84 + \frac{6}{AF - 3}\right] \qquad (3)$$

d. If the amplification factor is greater than 3.55 and the critical response peak is above the trip speed, the required separation margin (a percentage of maximum continuous speed) is equal to the following:

$$SM = \left[126 - \frac{6}{AF - 3}\right] - 100 \qquad (4)$$

2.8.2.6 The calculated unbalanced peak-to-peak rotor amplitudes (see 2.8.2.4, Item b) at any speed from zero to trip shall not exceed 75 percent of the minimum design diametral running clearances throughout the machine (with the exception of floating-ring seal locations). [2.8.2.7; 2.8.3.2; 2.8.3.3, Item b]

2.8.2.7 If, after the purchaser and the vendor have agreed that all practical design efforts have been exhausted, the analysis indicates that the separation margins still cannot be met or that a critical response peak falls within the operating speed range, acceptable amplitudes shall be mutually agreed upon by the purchaser and the vendor, subject to the requirements of 2.8.2.6.

2.8.3 SHOP VERIFICATION OF UNBALANCED RESPONSE ANALYSIS [2.8.2.4, ITEM D; 2.8.5.5; 4.3.3.3.3; 4.3.3.3.4]

2.8.3.1 A demonstration of rotor response at future unbalanced conditions is necessary because a well-balanced rotor may not be representative of future operating conditions (see 2.8.2.4, Item d). This test shall be performed as part of the mechanical running test (see 4.3.3), and the results shall be used to verify the analytical model. Unless otherwise specified, the verification test of the rotor unbalance shall be performed only on the first rotor (normally the spare rotor, if two rotors are purchased). The actual response of the rotor on the test stand to the same unbalanced weight as was used to develop the Bodé plots specified in 2.8.2.4 shall be the criterion for determining the validity of the damped unbalanced response analysis. To accomplish this, the following procedure shall be followed:

a. During the mechanical run test (see 4.3.3) the amplitudes and phase angle of the indicated vibration at the speed nearest the critical or criticals of concern shall be determined.
b. A trial weight, not more than one-half the amount calculated in 2.8.2.4, Item b, shall be added to the rotor at the location specified in 2.8.2.4, Item d; 90 degrees away from the phase of the indicated vibration at the speed or speeds closest to the critical or criticals of concern.
c. The machine should then be brought up to the operating speed nearest the critical of concern, and the indicated vibration amplitudes and phases shall be measured. The results of this test and the corresponding indicated vibration data from Item a above shall be vectorially added to determine the magnitude and phase location of the final test weight required to produce the required test vibration amplitudes.
d. The final test weight from Item c above shall be added to the rotor, and the machine shall be brought up to the operating speed nearest the critical of concern. When more than one critical of concern exists, additional test runs shall be performed for each, utilizing the highest speed for the initial test run.

Note: It is recognized that the dynamic response of the machine on the test stand will be a function of the agreed-upon test conditions and that unless the test-stand results are obtained at the conditions of pressure, temperature, speed, and load expected in the field, they may not be the same as the results expected in the field.

2.8.3.2 The parameters to be measured during the test shall be speed and shaft synchronous (1×) vibration amplitudes with corresponding phase. The vibration amplitudes and phase from each pair of *x-y* vibration probes shall be vectorially summed at each response peak to determine the maximum amplitude of vibration. The major-axis amplitudes of each response peak shall not exceed the limits specified in 2.8.2.6 (More than one application of the unbalance weight and test run may be required to satisfy these criteria.)

The gain of the recording instruments used shall be predetermined and preset before the test so that the highest response peak is within 60–100 percent of the recorder's full scale on the test-unit coast-down (deceleration; see 2.8.3.4). The major-axis amplitudes at the operating speed nearest the critical or criticals of concern shall not exceed the values predicted in 2.8.2.4, Item d, before coast-down through the critical of concern.

2.8.3.2.1 Vectorial addition of slow-roll (300—600 revolutions per minute) electrical and mechanical runout is required to determine actual vibration amplitudes and phase during the verification tests. Vectorial addition of the bearing-housing motion is required for machines that have flexible rotor supports (see 2.8.2.4, Item f).

Note 1: The phase on each vibration signal, x or y, is the angular measure, in degrees, of the phase difference (lag) between a phase reference signal (from a phase transducer sensing a once-per-revolution mark on the rotor, as described in API Standard 670) and the next positive peak, in time, of the synchronous (1x) vibration signal. (A phase change will occur through a critical or if a change in a rotor's balance condition occurs because of a shifting or looseness in the assembly.)

Note 2: The major axis amplitude is properly determined from a lissajous (orbit) display on an oscilloscope or equivalent instrument. When the phase angle between the X and Y signals is not 90 degrees, the major axis amplitude can be approximated by $(x^2 + y^2)^{1/2}$. When the phase angle between the x and y signals is 90 degrees, the major axis amplitude value is the greater of the two signals.

2.8.3.2.2 The results of the verification test shall be compared with those from the original analytical model. The vendor shall correct the model if it fails to meet the following acceptance criteria:

a. The actual critical speeds shall not deviate from the predicted speeds by more than ± 5 percent.
b. The predicted amplification factors shall not deviate from actual test-stand values by more than ± 20 percent.
c. The actual response peak amplitudes, including those that are critically damped, shall be within ± 50 percent of predicted amplitudes.

2.8.3.3 Additional testing is required if, from the test data described above or from the damped, corrected unbalanced response analysis (see 2.8.3.2.2), it appears that either of the following conditions exists:

a. Any critical response will fail to meet the separation margin requirements (see 2.8.2.5) or will fall within the operating speed range.
b. The requirements of 2.8.2.6 have not been met.

2.8.3.4 Rotors requiring additional testing per 2.8.3.3 shall receive additional testing as follows: Unbalance weights shall be placed as described in 2.8.2.4, Item b; this may require disassembly of the machine for placement of the unbalance weights. Unbalance magnitudes shall be achieved by adjusting the indicated unbalance that exists in the rotor from the initial run to raise the displacement of the rotor at the probe locations to the vibration limit defined by Equation 1 (see 2.8.2.4, Item b) at the maximum continuous speed; however, the unbalance used shall be no less than twice the unbalance limit specified in 2.8.5.2. The measurements from this test, taken in accordance with 2.8.3.2, shall meet the following acceptance criteria: [2.8.3.2]

a. At no speed outside the operating speed range, including the separation margins, shall the shaft deflections exceed 90 percent of the minimum design running clearances.
b. At no speed within the operating speed range, including the separation margins, shall the shaft deflections exceed 55 percent of the minimum design running clearances or 150 percent of the allowable vibration limit at the probes (see 2.8.2.4, Item b).

The internal deflection limits specified in Items a and b above shall be based on the calculated displacement ratios between the probe locations and the areas of concern identified in 2.8.2.4, Item c. Actual internal displacements for these tests shall be calculated by multiplying these ratios by the peak readings from the probes. Acceptance will be based on these calculated displacements or inspection of the seals if the machine is opened. Damage to any portion of the machine as a result of this testing shall constitute failure of the test. Minor internal seal rubs that do not cause clearance changes outside the vendor's new-part tolerance do not constitute damage.

4.3.3 MECHANICAL RUNNING TEST [2.8.3.1, 4.3.1.1]

4.3.3.1 The requirements of 4.3.3.1.1 through 4.3.3.1.12 shall be met before the mechanical running test is performed.

4.3.3.1.1 The contract shaft seals and bearings shall be used in the machine for the mechanical running test.

4.3.3.1.2 All oil pressures, viscosities, and temperatures shall be within the range of operating values recommended on the vendor's operating instructions for the specific unit being tested. For pressure lubrication systems, oil flow rates for each bearing shall be measured.

4.3.3.1.3 Test-stand oil filtration shall be 10 microns nominal or better. Oil system components downstream of the filters shall meet the cleanliness requirements of API Standard 614 before any test is started.

4.3.3.1.4 Bearings used in oil mist lubrication systems shall be prelubricated.

4.3.3.1.5 All joints and connections shall be checked for tightness, and any leaks shall be corrected.

4.3.3.1.6 All warning, protective, and control devices used during the test shall be checked, and adjustments shall be made as required.

4.3.3.1.7 Facilities shall be installed to prevent the entrance of oil into the compressor during the mechanical running test. These facilities shall be in operation throughout the test.

4.3.3.1.8 Testing with the contract coupling is preferred. If this is not practical, the mechanical running test shall be performed with coupling-hub idling adapters in place, resulting in moments equal (±10 percent) to the moment of the contract coupling hub plus one-half that of the coupling spacer. When all testing is completed, the idling adapters shall be furnished to the purchaser as part of the special tools. [3.2.4]

4.3.3.1.9 All purchased vibration probes, cables, oscillator-demodulators, and accelerometers shall be in use during the test. If vibration probes are not furnished by the equipment vendor or if the purchased probes are not compatible with shop readout facilities, then shop probes and readouts that meet the accuracy requirements of API Standard 670 shall be used.

4.3.3.1.10 Shop test facilities shall include instrumentation with the capability of continuously monitoring and plotting revolutions per minute, peak-to-peak displacement, and phase angle (*x-y-y´*). Presentation of vibration displacement and phase marker shall also be by oscilloscope.

4.3.3.1.11 The vibration characteristics determined by the use of the instrumentation specified in 4.3.3.1.9 and 4.3.3.1.10 shall serve as the basis for acceptance or rejection of the machine (see 2.8.5.5).

4.3.3.1.12 When seismic test values are specified, vibration data (minimum and maximum values) shall be recorded and located (clock angle) in a radial plane transverse to each bearing centerline (if possible), using shop instrumentation during the test.

4.3.3.2 Unless otherwise specified, the control systems shall be demonstrated and the mechanical running test of the equipment shall be conducted as specified in 4.3.3.2.1 through 4.3.3.2.6.

4.3.3.2.1 The equipment shall be operated at speed increments of approximately 10 percent from zero to the maximum continuous speed and run at the maximum continuous speed until bearings, lube-oil temperatures, and shaft vibrations have stabilized.

4.3.3.2.2 The speed shall be increased to 110 percent of the maximum continuous speed, and the equipment shall be run for a minimum of 15 minutes.

4.3.3.2.3 Overspeed trip devices shall be checked and adjusted until values within 1 percent of the nominal trip setting are attained. Mechanical overspeed devices shall attain three consecutive nontrending trip values that meet this criterion.

4.3.3.2.4 The speed governor and any other speed-regulating devices shall be tested for smooth performance over the operating speed range. No-load stability and response to the control signal shall be checked.

4.3.3.2.5 As a minimum, the following data shall be recorded for governors: sensitivity and linearity of relationship between speed and control signal, and adjustable governors, response speed range.

4.3.3.2.6 The speed shall be reduced to the maximum continuous speed, and the equipment shall be run for 4 hours.

Note: Caution should be exercised when operating equipment at or near critical speeds.

4.3.3.3 The requirements of 4.3.3.3.1 through 4.3.3.3.8 shall be met during the mechanical running test.

4.3.3.3.1 During the mechanical running test, the mechanical operation of all equipment being tested and the operation of the test instrumentation shall be satisfactory. The measured unfiltered vibration shall not exceed the limits of 2.8.5.5 and shall be recorded throughout the operating speed range.

4.3.3.3.2 While the equipment is operating at maximum continuous speed and at other speeds that may have been specified in the test agenda, sweeps shall be made for vibration amplitudes at frequencies other than synchronous. As a minimum, these sweeps shall cover a frequency range from 0.25 to 8 times the maximum continuous speed but not more than 90,000 cycles per minute (1500 Hertz). If the amplitude of any discrete, nonsynchronous vibration exceeds 20 percent of the allowable vibration as defined in 2.8.5.5, the purchaser and the vendor shall mutually agree on requirements for any additional testing and on the equipment's suitability for shipment.

4.3.3.3.3 The mechanical running test shall verify that lateral critical speeds conform to the requirements of 2.8.2 and 2.8.3. The first lateral critical speed shall be determined during the mechanical running test and stamped on the nameplate followed by the word test.

● **4.3.3.3.4** Plots showing synchronous vibration amplitude and phase angle versus speed for deceleration shall be made before and after the 4-hour run. Plots shall be made of both the filtered (one per revolution) and the unfiltered vibration levels, when specified. These data shall also be furnished in polar form. The speed range covered by these plots shall be 400 to the specified driver trip speed.

4.3.3.3.5 Shop verification of the unbalanced response analysis shall be performed in accordance with 2.8.3.

● **4.3.3.3.6** When specified, tape recordings shall be made of all real-time vibration data. [Item 32, Appendix C]

● **4.3.3.3.7** When specified, the tape recordings of the real-time vibration data shall be given to the purchaser.

● **4.3.3.3.8** Lube-oil and seal-oil inlet pressures and temperatures shall be varied through the range permitted in the compressor operating manual. This shall be done during the 4-hour test.

4.3.3.4 Unless otherwise specified, the requirements of 4.3.3.4.1 through 4.3.3.4.4 shall be met after the mechanical running test is completed.

4.3.3.4.1 Hydrodynamic bearings shall be removed, inspected, and reassembled after the mechanical running test is completed.

4.3.3.4.2 If replacement or modification of bearings or seals or dismantling of the case to replace or modify other parts to control mechanical or performance deficiencies, the initial test will not be acceptable, and the final shop tests shall be run after these replacements or corrections are made.

4.3.3.4.3 When spare rotors are ordered to permit concurrent manufacture, each spare rotor shall also be given a mechanical running test in accordance with the requirements of this standard.

4.3.3.4.4 After the mechanical running test is completed, each completely assembled compressor casing intended for toxic, hazardous, flammable, or hydrogen-rich service shall be tested as specified in 4.3.3.4.4.1 through 4.3.3.4.4.3. [4.3.4.8]

4.3.3.4.4.1 The casing (including end seals) shall be pressurized with an inert gas to the maximum sealing pressure of the maximum seal design pressure, as agreed upon by the purchaser and vendor; held at this pressure for a minimum of 30 minutes: and subjected to a soap-bubble test or another approved test to check for gas leaks. The test shall be considered satisfactory when no casing or casing joint leaks are observed.

4.3.3.4.4.2 The casing (with or without end seals installed) shall be pressurized to the rated discharge pressure, held at this pressure for a minimum of 30 minutes, and subjected to a soap-bubble test or another approved test to check for gas leaks. The test shall be considered satisfactory when no casing or casing joint leaks are observed.

4.3.3.4.4.3 The requirements of 4.3.3.4.4.1 and 4.3.3.4.4.2 may necessitate two separate tests.

SECTION 2—TRAIN TORSIONAL ANALYSIS

2.1 Introduction and Scope

Ensuring mechanical reliability in rotating machinery trains begins in the design phase of each train component. The vendor of each individual train component is called on to perform sufficient analysis of the proposed unit design such that, when finally constructed, the train component will perform reliably throughout its intended service life. When these components are combined into an equipment train, problems may arise that are different in nature from those the vendor must address during component design. Specifically, the torsional vibration behavior of the complete train is of importance in assuring that the individual units will reliably operate when coupled together. A diagram of a circular beam (shaft) undergoing a torsional deformation is presented in Figure 2-1. Torsional vibrations refer to oscillatory torsional deformations encountered by the shafts in the subject train during all phases of operation, including start-up. Note that this tutorial makes no attempt to address reciprocating compressor trains.

The typical approach to designing an equipment train from a torsional dynamics standpoint is to calculate the train's undamped torsional natural frequencies and attempt to locate them away from frequencies of potential excitation such as shaft operating speeds. By placing a train's torsional natural frequencies outside of the nominal design speed range on the 1× operating speed lines and away from any other known potential excitation frequencies (for example, electrical line frequency), the torsional natural frequencies will not cause operating problems such as coupling or shaft end torsional failure. When a torsional natural frequency interference with operating speed does occur, the designer calculates the coupling torsional stiffness that eliminates the interference and requests the design change from the coupling vendor. The coupling manufacturer can usually alter the coupling stiffnesses by at least ± 25 percent by redesigning the spool piece. Occasionally, however, an interference will occur where the natural frequency of an individual unit cannot be moved by a coupling stiffness change. When this happens, consideration must be given to modifying the individual units.

In general, experience indicates that control of torsional natural frequencies is easiest to accomplish when the coupling torsional stiffnesses are significantly smaller in magnitude than the local torsional stiffnesses of the unit shaft ends. When a train is designed in this manner, the torsional modes of motion can be separated into two categories: coupling controlled torsional modes (usually below the operating speed of the highest speed shafting in the train) and the shaft controlled modes (usually above the operating speed of the highest speed shafting in the train). An important advantage of this design philosophy is accurate calculation of the important lower frequency torsional modes, because the influence of assembly related variables (for example, shaft twisting in coupling hubs) is minimized.

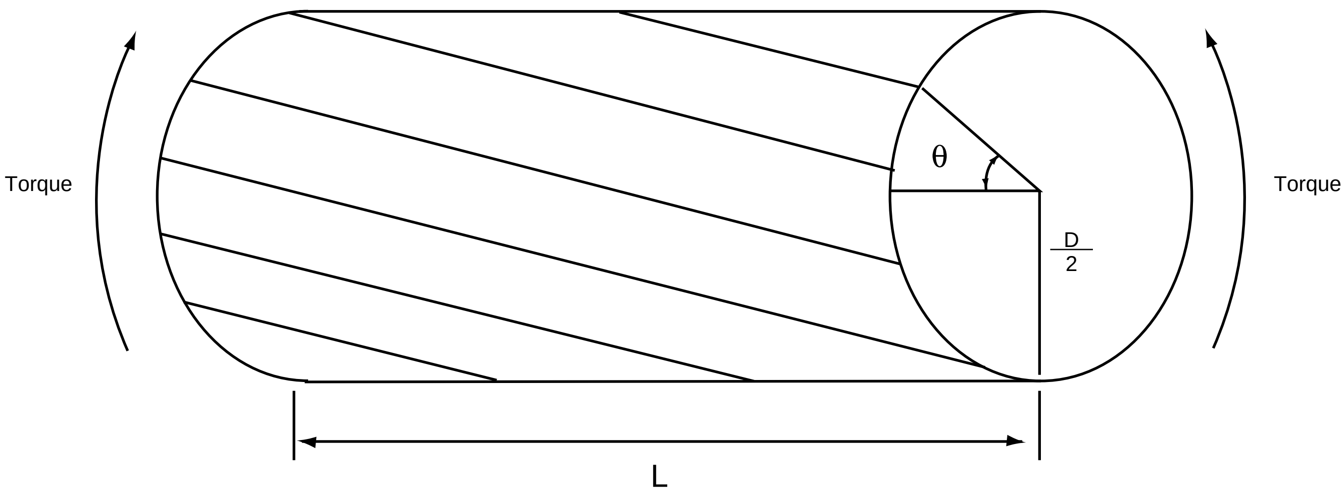
API provides a torsional analysis flow chart, presented in Figure 2-2, which is intended to aid in the design process of trains from a torsional dynamics standpoint. As indicated down the flow chart, certain requirements must be met, or redesign efforts are mandated. The essential requirements are that (a) torsional natural frequencies are 10 percent above or 10 percent below any possible excitation frequency within the operating speed range and (b) all nonsynchronous torsional excitations, such as 1× and 2× electric line frequency, have been identified. If these requirements are not met, then redesign efforts are required. In the special case where torsional natural frequencies are calculated to fall within the margin specified, and the purchaser and the vendor have agreed that all efforts to remove the critical from within the limiting frequency range have been exhausted, API mandates that a stress analysis shall be performed to demonstrate that the natural frequencies have no adverse effect on the complete train. Proper consideration of each of these steps will ensure that train torsional characteristics meet the requirements outlined by API and ultimately result in an equipment train that is free of torsional vibration problems.

This section outlines the methodology by which a train torsional analysis may be accomplished for the purpose of uncovering potential train torsional vibration problems. Correction of potential torsional vibration problems by redesign is emphasized in this tutorial. If it is not possible to remove a torsional natural frequency from the operating speed range, supplemental analysis (for example, damped response, fatigue, and so forth) may be used to determine, prior to train installation, if prolonged train operation will damage couplings or rotating elements. These supplemental analysis are described in this tutorial.

2.2 Purpose of the Train Torsional Analysis

Any new design or significant design modification in a machine's rotating system, including its impellers, couplings, gears, and drivers, typically requires an undamped train torsional natural frequency speed analysis. This analysis is necessary to determine the placement of the torsional natural frequency frequencies relative to operating speeds and whether this meets with acceptance criteria set forth by API and by generally accepted industry standards.

The actual requirements for a train torsional vibration analysis are specified by API Standard Paragraph 2.8.4.5.



For a solid isotropic cylinder,

$$\theta = \frac{TL}{GJ}$$

θ = Angle of twist
T = Applied torque
L = Cylinder length
G = Shear modulus
J = Second moment of area of cross section about the axis of twist

$$= \frac{\pi D^4}{32} \quad \text{(Circular cross-section)}$$

Figure 2-1—Diagram of a Shaft Undergoing Torsional Elastic Deflection

This requirement alludes to the more common sources of torsional excitation or torque pulsations in machinery, as in the following examples:

a. Electrical frequency excitation in motors and generators.
b. Gear problems such as pitch line runout, tooth profile errors, poor gear tooth finish, and so forth.
c. Fluid dynamic pulsations.
d. Coupled lateral-torsional interaction.
e. Other known sources of pulsations that are characteristic of specific machinery by virtue of experience.

When one of these sources of torsional excitation coincides with an undamped torsional natural frequency, the torsional mode of vibration may become amplified and potentially lead to immediate damage or longer-term fatigue damage. The customary undamped train torsional natural frequency analysis is intended to ensure that these coincidences are avoided and that torsional vibration characteristics will not lead to premature machinery failure. In most cases, the critical speed analysis will be sufficient to ensure acceptable torsional vibration characteristics; however, in special circumstances additional analysis may be required. For instance, if a torsional interference cannot be removed by standard methods of redesign, a stress analysis may be required to ensure that the interference will not adversely affect the machinery train. For other cases, such as a train with a synchronous motor driver, a transient torsional vibration analysis is usually required.

2.3 API *Standard Paragraphs*

The API *Standard Paragraphs*, (R20) concerning the torsional analysis of machinery trains is provided in Appendix 2A. Several aspects of this specification will be discussed further in subsequent sections.

2.4 Basic Analysis Method

A coupled train's torsional characteristics are determined by calculating the train's undamped torsional natural frequencies and mode shapes. In this analysis, computer models of the individual rotors (data provided by vendors) are linked using simplified stiffness and inertia models of the couplings. The coupling data required for the analysis (coupling and torsional stiffness and inertia) can be either directly supplied by the vendors or calculated using information provided in vendor supplied catalogues. Note that WR^2 and Ipg are often interchangeable, $WR^2 = Ipg$. Where g = gravita-

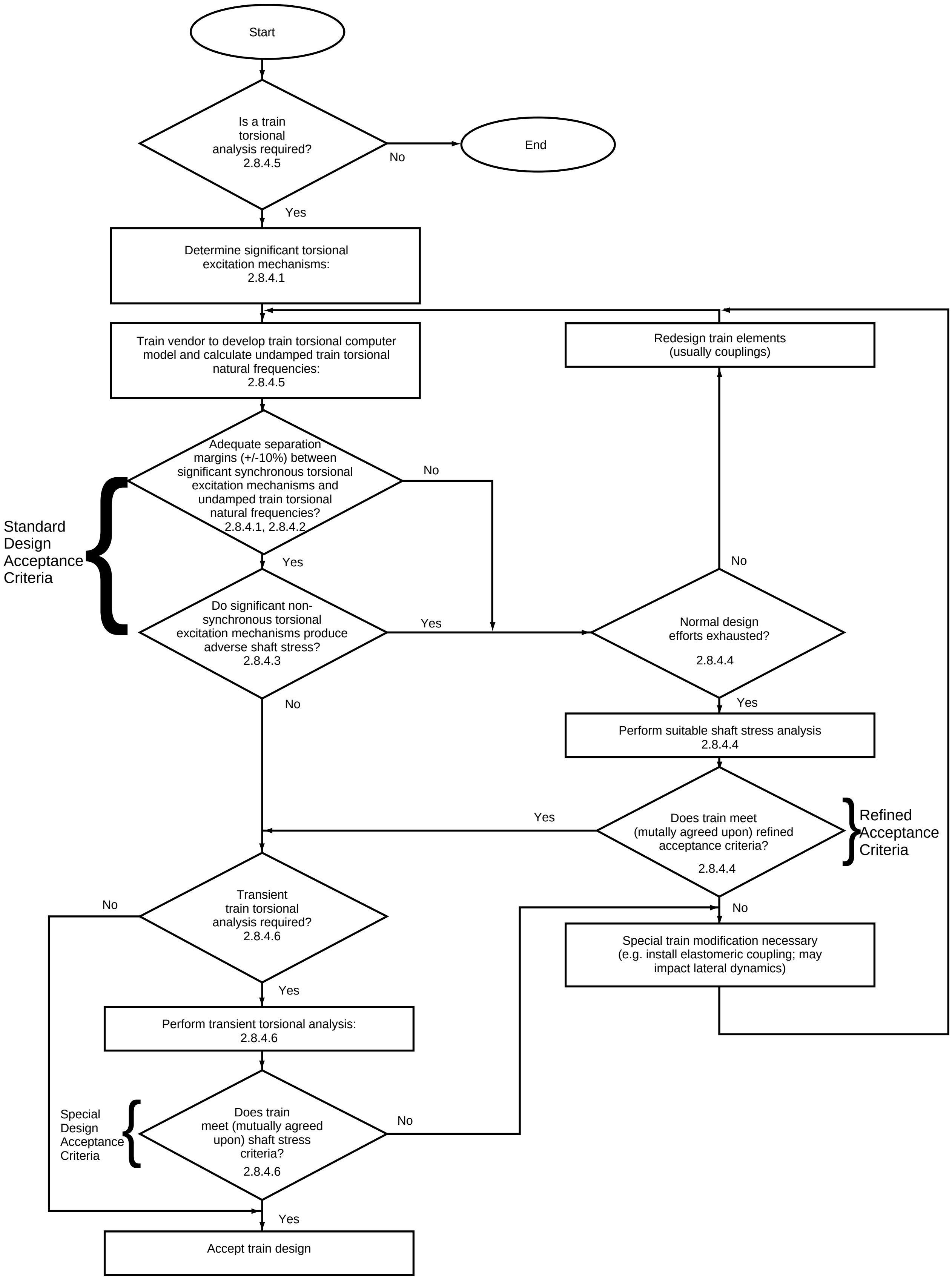


Figure 2-2—Rotor Dynamics Logic Diagram (Torsional Analysis)

tional acceleration. The undamped torsional natural frequency analysis assumes all individual train elements, including rotors and couplings, are linear elastic elements. Thus, nonlinear effects such as gear backlash and mechanical looseness are neglected. Finally, torsional damping is neglected in this analysis since the levels of actual torsional system damping are low.

Several types of computer programs are available for calculating the undamped torsional natural frequencies. Such software will calculate the undamped torsional modes using the Finite Element Method or the Transfer Matrix (Holzer) Method. Such codes should be properly validated and be sufficiently robust to analyze all systems of interest to the user. For convenience to the reader, some computational aspects of the Transfer Matrix (Holzer) Method are detailed in Appendix 2B as well as its limitations in predicting undamped torsional modes.

2.5 Discussion of Train Modeling

Great care must be exercised in the detailed modeling of a torsional system in order to ensure the required accuracy. A consistent set of engineering units must be used throughout the analysis in order to avoid potential errors. This is especially true of torsional analysis work where several sets of vendor prints and/or data sets with different units systems may be present.

Train modeling begins by dividing the component shafts into discrete sections or finite elements subject to the following:

a. End shaft sections at step changes in either OD or ID of the shaft.
b. The length-to-diameter ratio of any section should not exceed 1.0.
c. The length-to-diameter ratio of any section should not be less than 0.05.
d. The number of shaft sections used to generate a model of the train should be minimized where possible.

If train units are accurately modeled, the undamped torsional natural frequency analysis usually predicts actual train natural frequencies within a small margin of error because most equipment trains generally possess low levels of system torsional damping.

Figures 2-3 through 2-8 present, in sequence, the general approach to torsional modeling for a typical motor-gear-compressor train and a turbine-compressor train. Side view drawings of the two trains are presented in Figures 2-3 and 2-6. Cross-sectional views of the rotating elements for these two trains are displayed in Figures 2-4 and 2-7. Finally, using geometric and inertia properties of the rotating elements, computer models of the trains can be assembled. Schematics of the train computer models are presented in Figures 2-5 and 2-8.

Typical items which can easily be modeled by concentrated mass-elastic data exclusively are couplings, gears, impellers, turbine stages, and motor rotor attachments. Experience has shown that flexible couplings are most appropriately modeled as a single torsional spring (vendor supplied torsional stiffness) with the respective half-coupling inertias at each end. Gears lend themselves to lumped mass modeling since the bulk of their inertia is in the gear wheels, and the shafts closely approximate low inertia torsional springs. Impellers and turbine stages can usually be modeled as discrete lumped inertias since they typically do not contribute to the torsional stiffness of their respective shafts. Motor rotor attachments such as rotor cores and brushless exciters are not easily modeled because they are typically shrunk onto the shaft over an extended length, and it is not obvious how their inertias and stiffnesses enter the motor shaft. Caution must be exercised with approaches that lump the inertia and stiffness of a whole unit because inaccuracies may result if the shaft ends are sufficiently torsionally flexible relative to couplings.

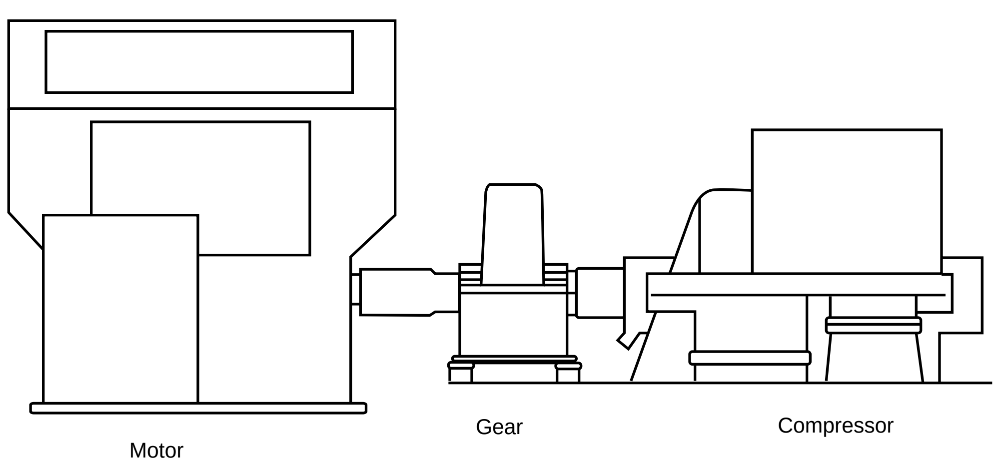
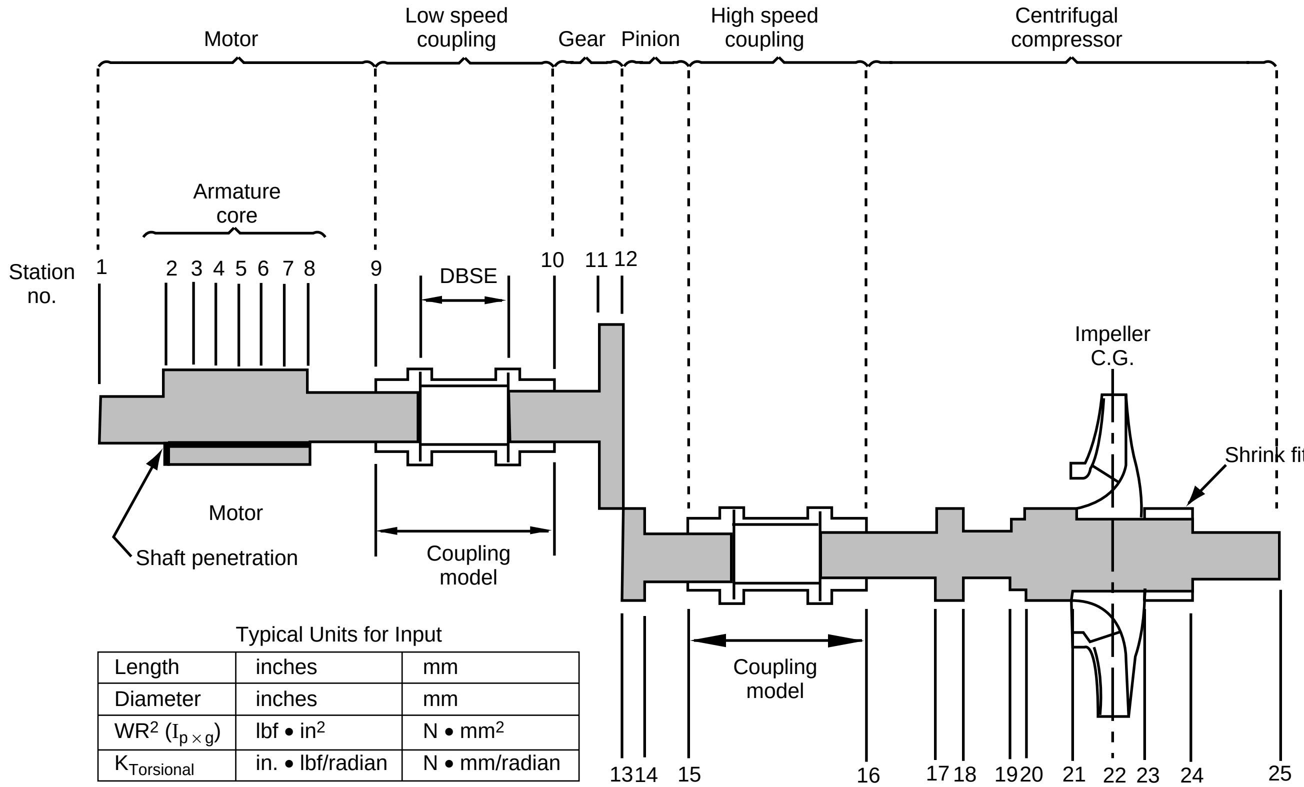


Figure 2-3—Side View of a Typical Motor-Gear-Compressor Train



Typical Units for Input

Length	inches	mm
Diameter	inches	mm
WR^2 ($I_{p \times g}$)	lbf • in^2	N • mm^2
$K_{Torsional}$	in. • lbf/radian	N • mm/radian

Note: Coupling vendors typically provide WR^2 and $K_{Torsional}$ for each coupling. The WR^2 value does not include the WR^2 of the coupling journal (shaft inside the coupling HUB). The $K_{Torsional}$ value typically assumes 1/3 shaft penetration into the coupling HUB.

Figure 2-4—Modeling a Typical Motor-Gear-Compressor Train

2.6 Specific Modeling Methods and Machinery Considerations

This section presents some of the more important modeling problems and concerns typically encountered in torsional vibration analysis:

a. Speed referencing inertia (I_p) and stiffness (K_t.).
b. Step shafting.
c. Shrink fits.
d. Integral disks or hubs.
e. Couplings.
f. Speed increasing or decreasing gears.
g. Electric motors and generators.
h. Analysis of pumps.
i. Other considerations.

2.6.1 SPEED REFERENCING INERTIA (I_P) AND STIFFNESS (K_t)

A computer code for calculating undamped train torsional natural frequencies should have the capability to analyze a train of coupled rotors that operate at different rotative speeds (for example, an equipment train with multiple speed-changing gears). In order to formulate an equivalent single shaft model for a geared train (required before calculating the undamped torsional natural frequencies and mode shapes of the system), all inertias and stiffnesses must be referenced to a common speed. The relationship between inertias and stiffnesses of units that operate at a different speed from a selected reference speed is written as follows:

$$I_{pr} = I_{pa} \times \left(\frac{N_a}{N_r}\right)^2 \tag{2-1}$$

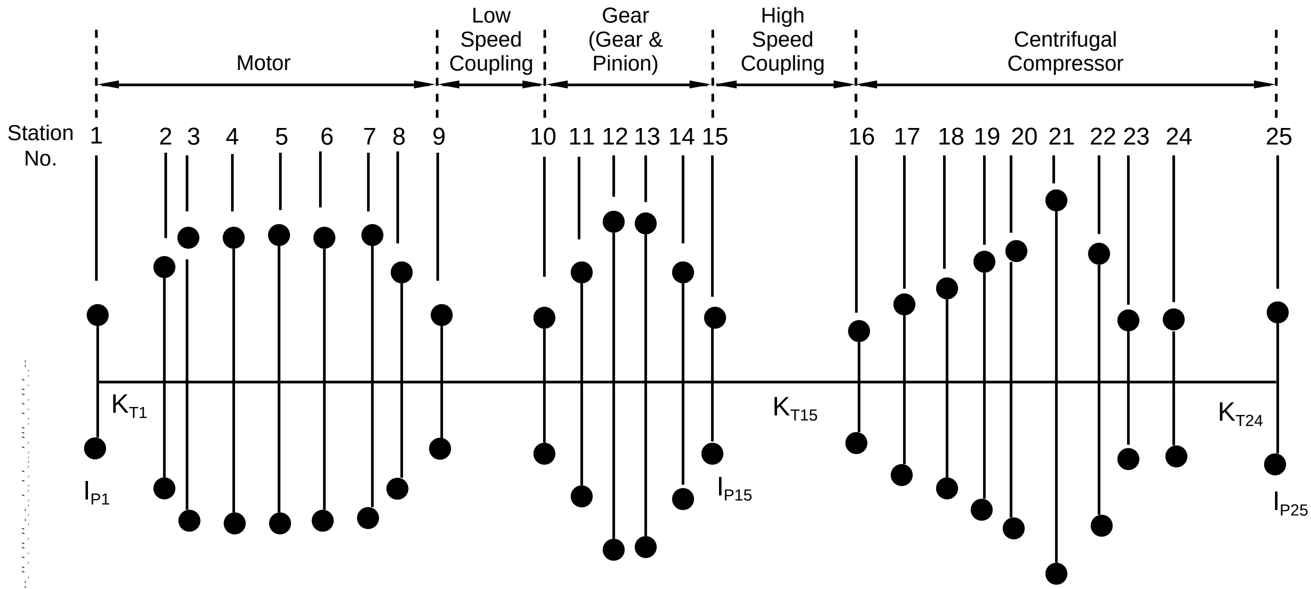
$$K_{tr} = K_{ta} \times \left(\frac{N_a}{N_r}\right)^2 \tag{2-2}$$

Where:

Quantity	Typical SI Units	Typical US Customary Units
I_p = polar mass moment of inertia	kg-m^2	lb-in^2
K_t = torsional stiffness	N-m/rad	in-lb/rad
N = rotation speed	rev/m	rev/m

Note:
Subscript *a* denotes *actual*.
Subscript *r* denotes *reference*.

If the software used to calculate the undamped torsional natural frequencies does not have the capability to analyze a



Notes:

1. I_{Pi} = ith station lumped polar moment of inertia.
   K_{Ti} = ith shaft section torsional stiffness.
2. All WR^2 values have been converted into polar mass moments of inertia (I_P) at each station.
3. All I_P and K_T values have been referenced to a single shaft rotation speed.

Figure 2-5—Schematic of Lumped Parameter Train Model for Torsional Analysis

train whose elements are operating at different speeds, then the engineer must manually perform speed referencing using the above relationships.

Simple gear reductions (single or multiple) represent single-branch systems, and combined with possible simple branches (single inertia, one-degree of freedom) can be analyzed with the basic Transfer Matrix (Holzer) computer code. Multiple-branch systems of greater complexity require a more sophisticated computer analysis. A side view drawing of a train with a single reduction gear is displayed in Figure 2-3. An example of a multiple branch system, a multi-stage integrally geared plant air compressor, is displayed in Figure 2-9. This unit has four overhung compressor stages driven through a single bull gear. Note that the two pinions operate at different speeds.

2.6.2 STEP SHAFTING

When the shaft geometry length (L) and diameter (D), is used as input, the computer code should consider the effective penetration of smaller diameter shaft sections into adjacent larger diameter sections. The smaller shaft, in effect, penetrates the larger by the amount penetration factor (PF) so that the length of the smaller diameter shaft is effectively increased by the amount PF and the length of the larger diameter shaft is reduced by the same amount. Figure 2-10 displays the effective length increase of the smaller diameter section as a function of the step geometry. Allowance for penetration effects enables one to more accurately approximate the actual flexibility of the physical system than by simply summing the calculated flexibilities of the individual shaft sections. For this type of discontinuity, the effective length of a shaft which joins another of larger diameter is greater than the actual length due to local deformation at the juncture. As shown in Figure 2-10, this penetration factor (PF) depends on the ratio of shaft diameters and can be determined by the following equation.[1]

$$L_e = \left[\frac{L_1 + PF}{D_1^4} + \frac{L_2 - PF}{D_2^4}\right] D_e^4 \qquad (2\text{-}3)$$

Where:

Quantity	Typical SI Units	Typical US Customary Units
L_e = effective length	millimeters	inches
D_e = effective diameter	millimeters	inches
PF = penetration factor	dimensionless	dimensionless

[1]W. Ker Wilson, *Practical Solution of Torsional Vibration Problems*, Wiley & Sons, Inc., New York, New York, 1956.

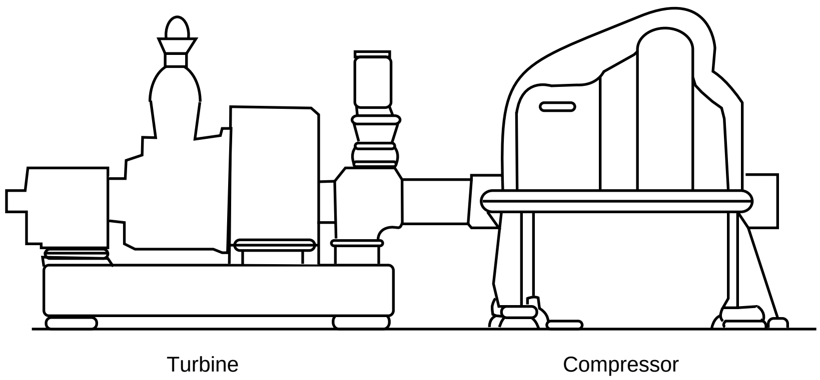
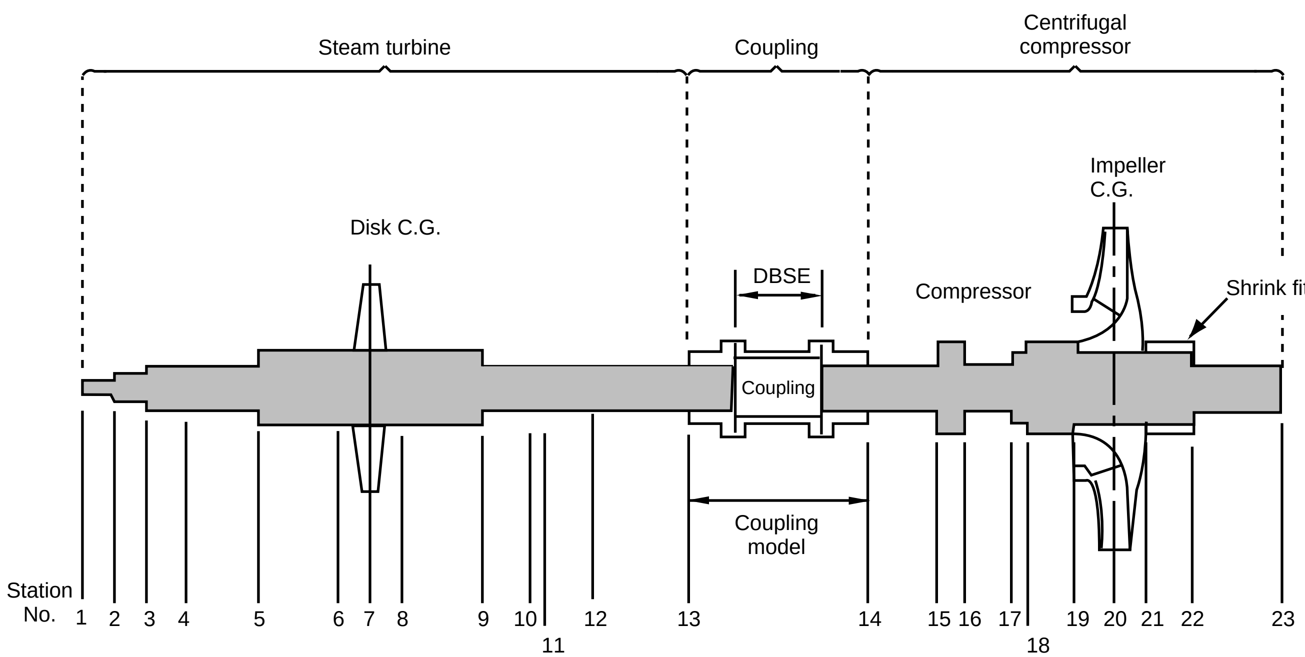


Figure 2-6—Side View of a Typical Turbine-Compressor Train

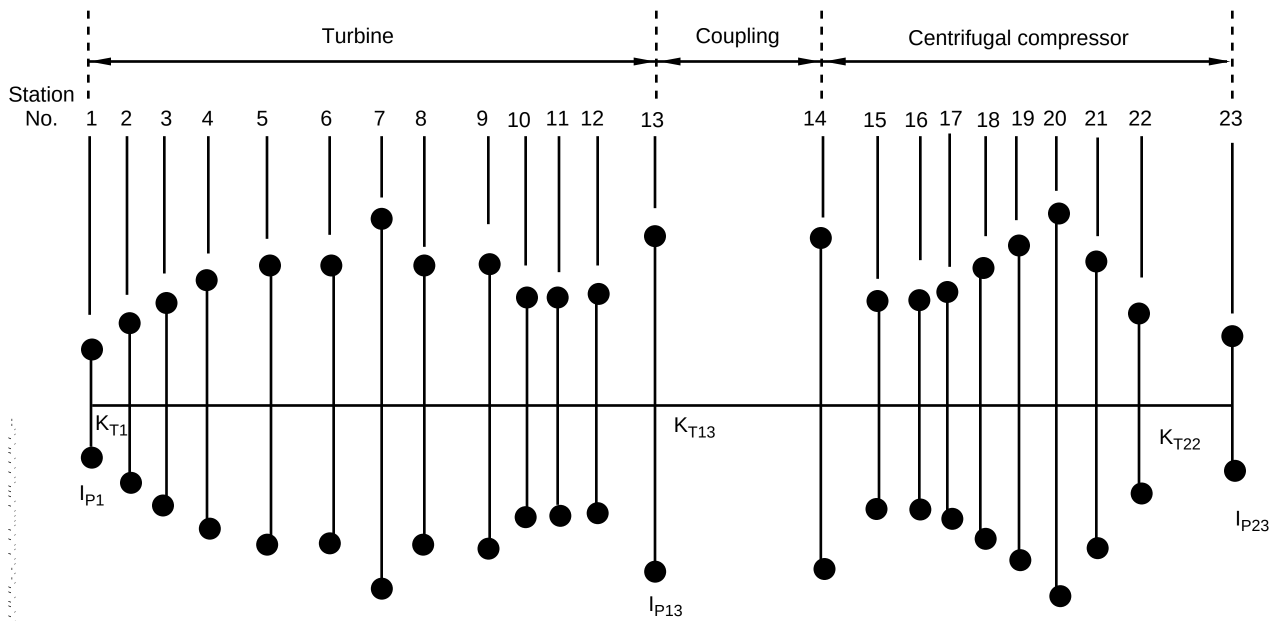


Typical Units for Input

Length	inches
Diameter	inches
WR^2 ($I_p \times g$)	lbf • in^2
$K_{Torsional}$	in. • lbf/radian

Note: Coupling vendors typically provide WR^2 and $K_{Torsional}$ for each coupling. The WR^2 value does not included the WR^2 of the coupling journal (shaft inside the coupling HUB). The $K_{Torsional}$ value typically assumes 1/3 shaft penetration into the coupling HUB.

Figure 2-7—Modeling a Typical Turbine-Compressor Train



Notes:
1. I_{Pi} = ith station lumped polar moment of inertia.
K_{Ti} = ith shaft section torsional stiffness.
2. All WR^2 values have been converted into polar mass moments of inertia (I_P) at each station.
3. All I_P and K_T values do not require speed referencing since train elements all spin at the same speed.

Figure 2-8—Schematic of Lumped Parameter Train Model for Torsional Analysis

Table 2-1 gives some characteristic results.

Table 2-1—Penetration Factors for Selected Shaft Step Ratios

D_2/D_1	PF/D_1
1.00	0
1.25	0.055
1.50	0.085
2.00	0.100
3.00	0.107
∞	0.125

2.6.3 SHRINK FITS

Most rotating assemblies used in machinery trains have non-integral collars, sleeves, and so on, that are shrunk onto the shaft during rotor assembly. These shrunk-on components may or may not contribute to the torsional stiffness of the shaft, depending on amount and length of the shrink fit and size of the shrunk-on component. Precise guidelines regarding inclusion or exclusion of the effect of the shrunk-on member on shaft torsional stiffness are difficult to quantify; however, the following general principles apply:

a. If the fit of the shrunk-on component is relieved over most of its length, then the torsional stiffening effect is negligible. A fit is relieved when some portion of the designed interference has been removed. Schematics of shaft sleeves with and without typical relieved fits are displayed in Figure 2-11. Sleeves and impellers often possess relieved fits to aide the rotor assembly process and to minimize internal friction forces that contribute to rotor system instability. The stiffening effect of shaft sleeves and impellers with a high degree of relief (small fit length) is often neglected.

b. If the fit of a shrunk-on component is (1) *not* relieved over a significant part of its length, (2) made of the same material as the shaft, and (3) manufactured with a shrink fit equal to or greater than 1 mil/inch of shaft diameter, then the effective stiffness diameter of the shaft should be assumed equal to the actual diameter under the sleeve plus the thickness of the sleeve.[1]

c. If the shrunk-on component has a large rotational inertia, then centrifugal loading may diminish the effect of the shrink fit and the attendant torsional stiffening effects of the component over the rotor operating speed range.

d. If a shrunk on component has a nominal fit length with L/D greater than or equal to 1, the shaft is assumed to be unrestrained by the hub over an axial length one-third of the total length of shrunk surface. This ⅓ penetration is typically used by coupling vendors when specifying coupling torsional stiffness from hub to hub.

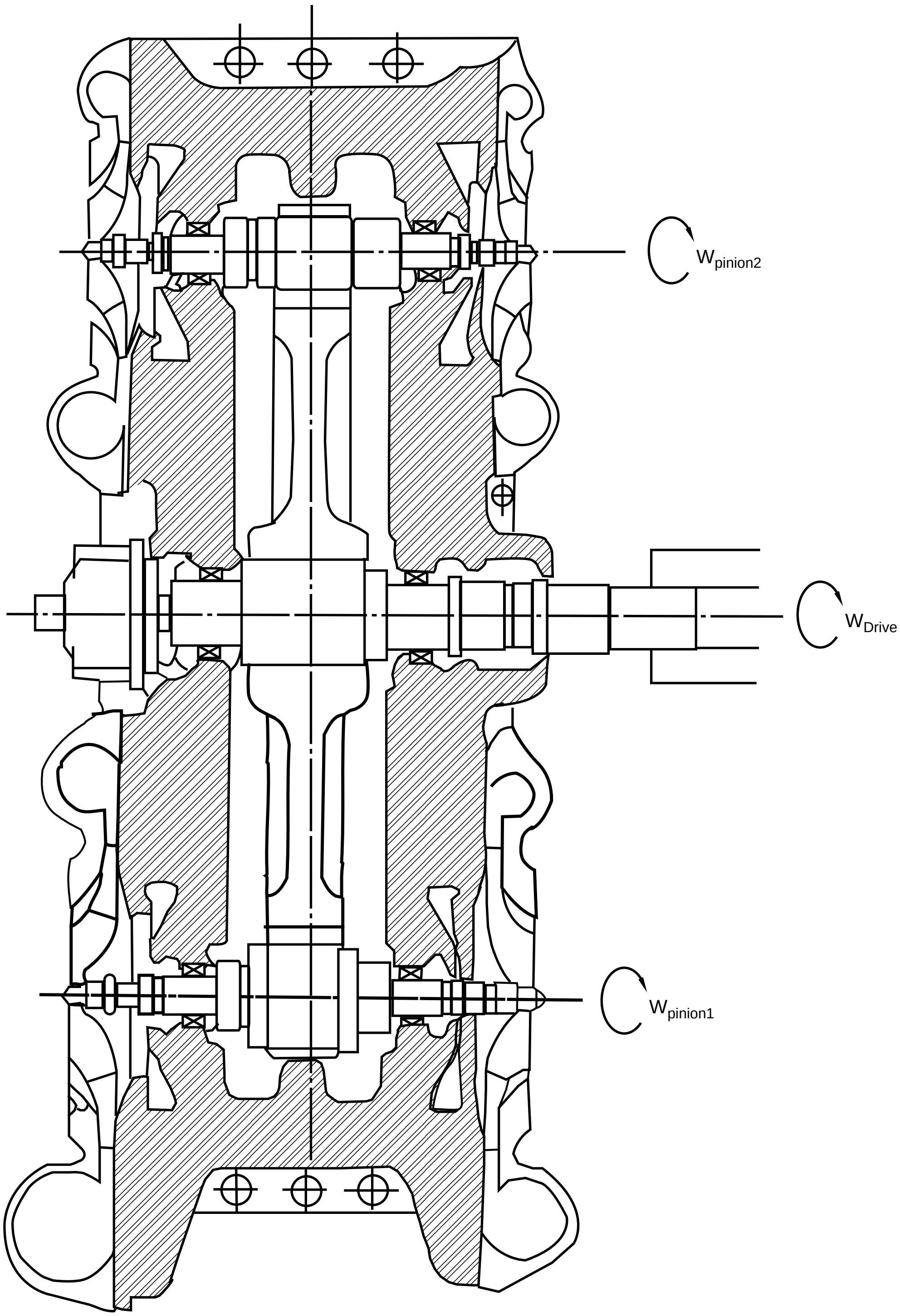


Figure 2-9–Multi-Speed Integrally Geared Plant Air Compressor
(A Multiple Branched System)

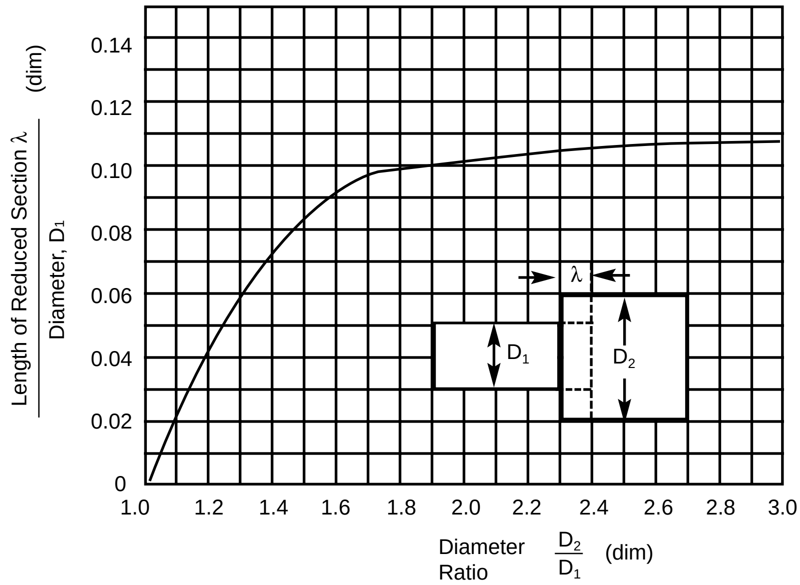


Figure 2-10—Effective Penetration of Smaller Diameter Shaft Section Into A Larger Diameter Shaft Section Due To Local Flexibility Effects (Torsion only)

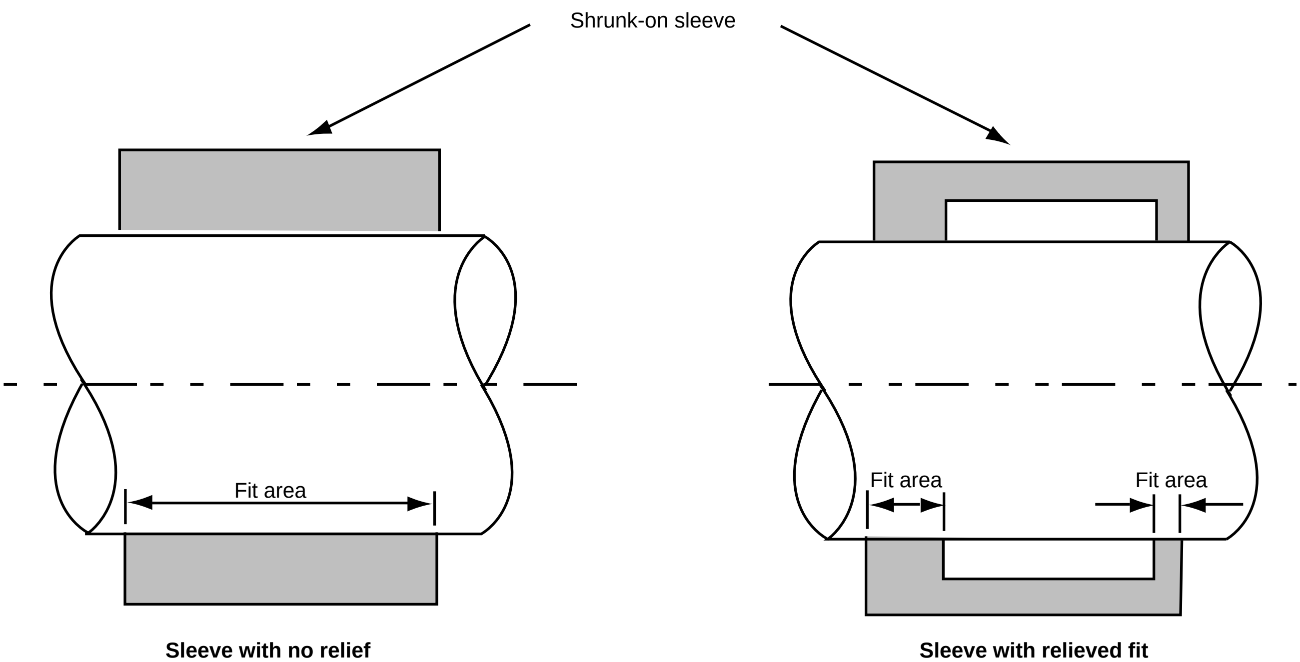


Figure 2-11— Examples of Shrunk-On Shaft Sleeves With and Without Relieved Fits

2.6.4. INTEGRAL DISKS OR HUBS

The effect of integral thrust collars or disks forged on the shaft can be determined by the method used for stepped shafts. For short collars (axial length less than ¼ the shaft diameter), the effect is negligible, and the length occupied by the collar may be assumed to have an effective torsional stiffness diameter equal to the diameter of the shaft.

2.6.5. COUPLINGS

The two types of couplings that normally connect train components are lubricated gear couplings and flexible element dry couplings. Sample cross-sectional drawings of these types of couplings are displayed in Figure 2-12. Normally a vendor-supplied torsional stiffness value is input for the total coupling. This value should include an assumed penetration factor (*PF*). Figure 2-10 displays the effective decrease in torsional rotor stiffness that results from penetration when a step change in shaft diameter is present. The concept of penetration through a step applies to couplings where a hub is shrunk onto a shaft without relief. The traditional starting assumption is that $PF = 1/3$. Stated simply, the shaft is considered unrestrained by the coupling hub for a distance of ⅓ of the length of the fit, starting at the coupling hub edge opposite the shaft end. Actual penetration of the shaft into the coupling hub will vary with the design of the coupling. For example, keyed hubs will possess a different penetration from hydraulic fit hubs. An accurate mathematical description of the coupling hub *PF* is crucial to generating an accurate torsional model. Several coupling manufacturers provide experimental and/or empirical data to provide a more accurate description of *PF* (and thus coupling total torsional stiffness, K_t). The empirical corrections often result in $PF > 1/3$ and suggest a looser hub-to-shaft fit than is typically assumed. Thus, some torsional natural frequencies may be lower than those predicted using a $PF = 1/3$.

Two types of couplings in common use today in turbomachinery design are the following:

a. Lubricated gear type couplings.
b. Non-lubricated *dry* flexible element couplings.

A gear coupling may be thought of as an assembly of torsional springs in series:

a. Hub-to-shaft connection stiffnesses (including shaft penetration into the hub).
b. Hub to sleeve assembly stiffnesses, (including hubs, sleeves, bolts, and so on).
c. Spool or spacer piece stiffness.

The total coupling torsional stiffness is expressed in the following equation:

$$K_t\,\text{COUPLING} = \frac{1}{\dfrac{1}{K_{c1}} + \dfrac{1}{K_{a1}} + \dfrac{1}{K_s} + \dfrac{1}{K_{c2}} + \dfrac{1}{K_{a2}}} \quad (2\text{-}4)$$

Where:

Quantity	Typical SI Units	Typical US Customary Units
K_{c1}, K_{c2} = hub connection stiffness	N-m/rad	in.-lb/rad
K_{a1}, K_{a2} = hub assembly stiffness	N-m/rad	in.-lb/rad
K_s = spacer stiffness	N-m/rad	in.-lb/rad

Note that some gear coupling assemblies possess short, stiff spacer tubes, and the hub-to-shaft connection stiffnesses can be significant. In such cases the accuracy of calculated coupling torsional stiffness is dependent on the accuracy of the *PF*. Conversely, inaccuracies of modeling shaft-to-hub penetration effects are minimized in gear couplings with a low total torsional stiffness due to long spacers.

The torsional stiffness of a flexible element coupling is calculated in a manner similar to that for a gear coupling with an additional torsional spring element added to account for the stiffness of the flexible element. For a given application, flexible element couplings tend to be torsionally softer than their gear-type counterpart, and the effects of *PF* less significant.

Elastomeric couplings are sometimes used to add torsional damping to equipment trains. The additional torsional damping helps attenuate the torsional vibrations that accompany the interference of a torsional natural frequency with a torsional excitation mechanism. Such couplings are used only when all other attempts to remove the natural frequency have been exhausted. This coupling can be heavier and possesses greater potential for unbalance than the dry or lubricated coupling it replaces and may introduce lateral rotor dynamics problems into the connected units. This type of coupling is also more maintenance intensive than an equivalent gear or flexible element dry coupling and may require replacement under prolonged operation. Even prior to coupling replacement, the material properties of the elastomeric element may change with time so the desired stiffness and damping characteristics may likewise vary.

2.6.6. SPEED INCREASING OR DECREASING GEARS

Simple gear reductions (single or double) represent single-branch systems that can be analyzed using the basic Transfer Matrix (Holzer) torsional method. As displayed in Equations 2-1 and 2-2, formulation of an equivalent single shaft model requires that all inertias and stiffnesses be referenced to the reference speed by the square of the gear ratio. In modeling the shaft stiffness characteristics of gears (both integral and shrink-fit gear construction), it is necessary to consider penetration of the shaft into the gear mesh. The rotational inertias of the gear and pinion are normally given on the drawings supplied by the manufacturer and are typically referenced to their own respective speeds. The torsional

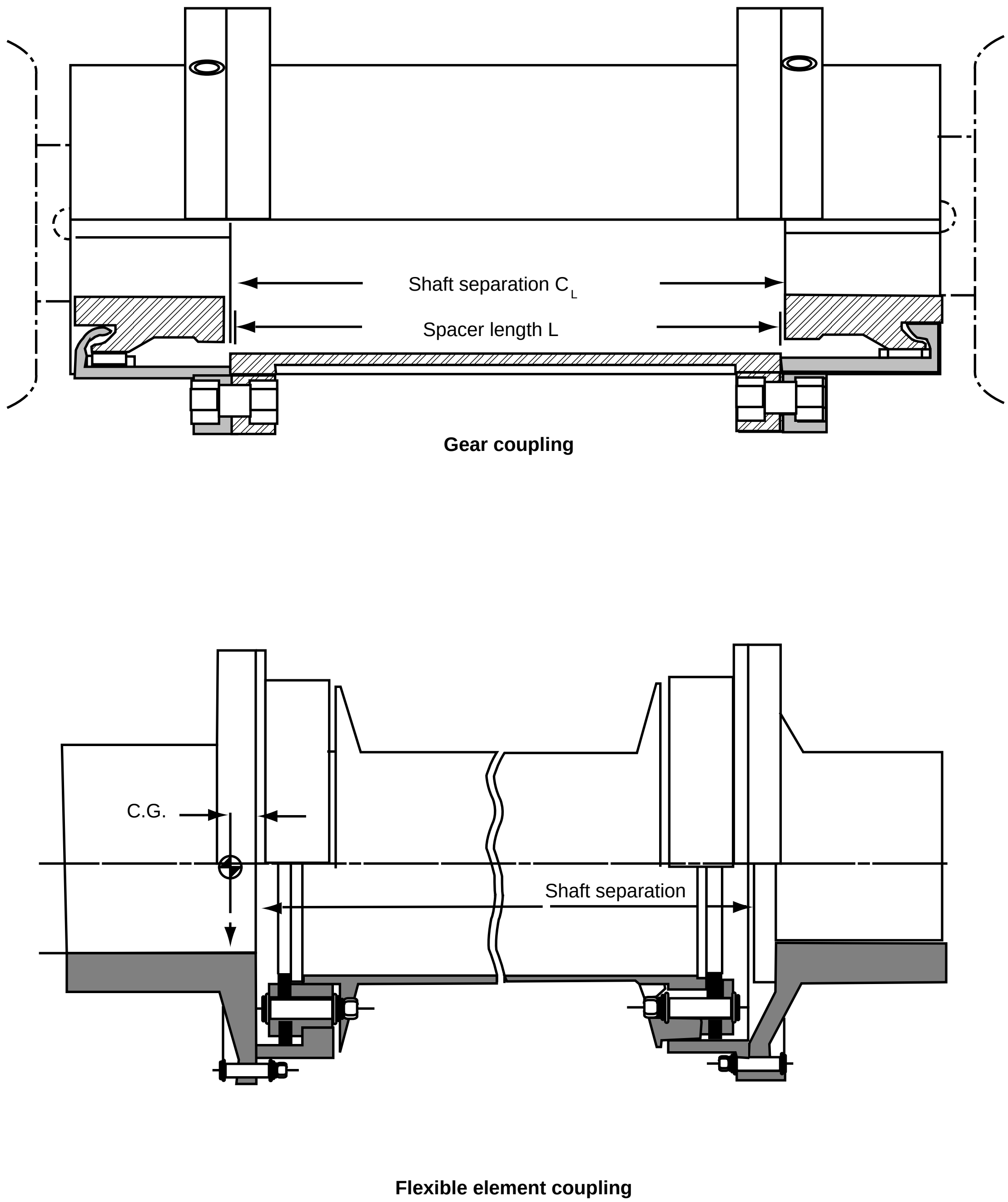


Figure 2-12–Cross-Sectional Views of Gear and Flexible Element (Disk-Type) Couplings

model of the gear will include stiffnesses and inertias of the bull gear up to the centerline of the gear; then the model will include the pinion's stiffnesses and inertias from the centerline to the end of the pinion. The inertia of the second half of the bull gear and the first half of the pinion are lumped at the centerline of the gear (see Figures 2-13 and 2-14). These two inertias are coupled in the gearbox through the torsional stiffness generated by the gear mesh. The gear mesh torsional stiffness is calculated using Equation 2-5 if the reference speed equals the pinion rotation speed, or Equation 2-6 if the reference speed equals the gear rotation speed:

$$K_{t\,\mathrm{MESH}} = 0.02725\ W_f\ PD_P^2 \cos^2\psi\left(\frac{E}{2}\right) \tag{2-5}$$

$$K_{t\,\mathrm{MESH}} = 0.02725\ W_f\ PD_G{}^2\ \cos^2\psi\left(\frac{E}{2}\right) \tag{2-6}$$

Where:

Quantity	Typical SI Units	Typical US Customary Units
K_{mesh} = mesh torsional stiffness	N-m/rad	in.-lb/rad
W_f = face width of mating gears	meters	inches
PD_P = pitch diameter of pinion	meters	inches
PD_G = pitch diameter of bull gear	meters	inches
ψ = helix angle of gear set	degrees	Degrees
E = Young's modulus	N/m²	lb/in²

2.6.7 ELECTRIC MOTORS AND GENERATORS

Precise, detailed torsional models of electric motors must be developed for inclusion in the train model if some of the train torsional modes are to be accurately calculated. For example, in motor-gear-compressor trains, the third torsional mode is almost exclusively governed by the stiffness and inertia characteristics of the motor core. Rotating motor exciters can also add an additional independent frequency to the torsional system. This mode will be inaccurate or totally missed without a finely divided motor rotor model.

For the purpose of torsional modeling, motors can be divided into two groups: those with spiders or webs attached to the base shaft to support the motor core and those that do not possess such construction. For the purpose of this document, electric machinery designed without the spider or web arms is said to possess *laminated construction*. Despite their simple appearance, motors with laminated construction are difficult to accurately model because the contribution of the shrunk-on core to the motor's midspan shaft stiffness is difficult to analytically predict. The torsional/shear stress paths in the area of the motor core are complex and highly dependent on the exact magnitude of the interference fits. Tolerances in these fits may alter the depth of the effective base shaft penetration into the motor core and may substantially change the torsional stiffness characteristics of the motor.

Accurate modeling of webbed motors requires the following:

a. The inertia of the rotating armature or poles must be distributed along the axial length of the core. Both the inertia of the rotating armature and the base shaft should be incorporated into the model.
b. The stiffening effect of the web arms must be added to the base shaft.

Calculation of the torsional stiffness of non-circular cross-sections such as the webbed midspan area of an electric motor is a complicated problem. A schematic of a typical webbed motor cross section is presented in Figure 2-15. Although numerous analytical approaches have been used to date, including detailed finite element techniques, the following approximate method has been used successfully (yielding good agreement upon correlation with test results). Non-circular section torsional rigidity can be satisfactorily approximated from equation:[2]

$$K_a = \frac{A^4}{4}\pi^2 I_p \tag{2-7}$$

Where:

Quantity	Typical SI Units	Typical US Customary Units
A = cross sectional area of the shaft including ribs	mm²	in.²
IP = polar moment of inertia	kg-m²	lb-in.²
K_a = approximate torsional stiffness of the non-circular cross section	N-m/rad	in.-lb/rad
K_e = exact torsional stiffness of the non-circular cross section	N-m/rad	in.-lb/rad

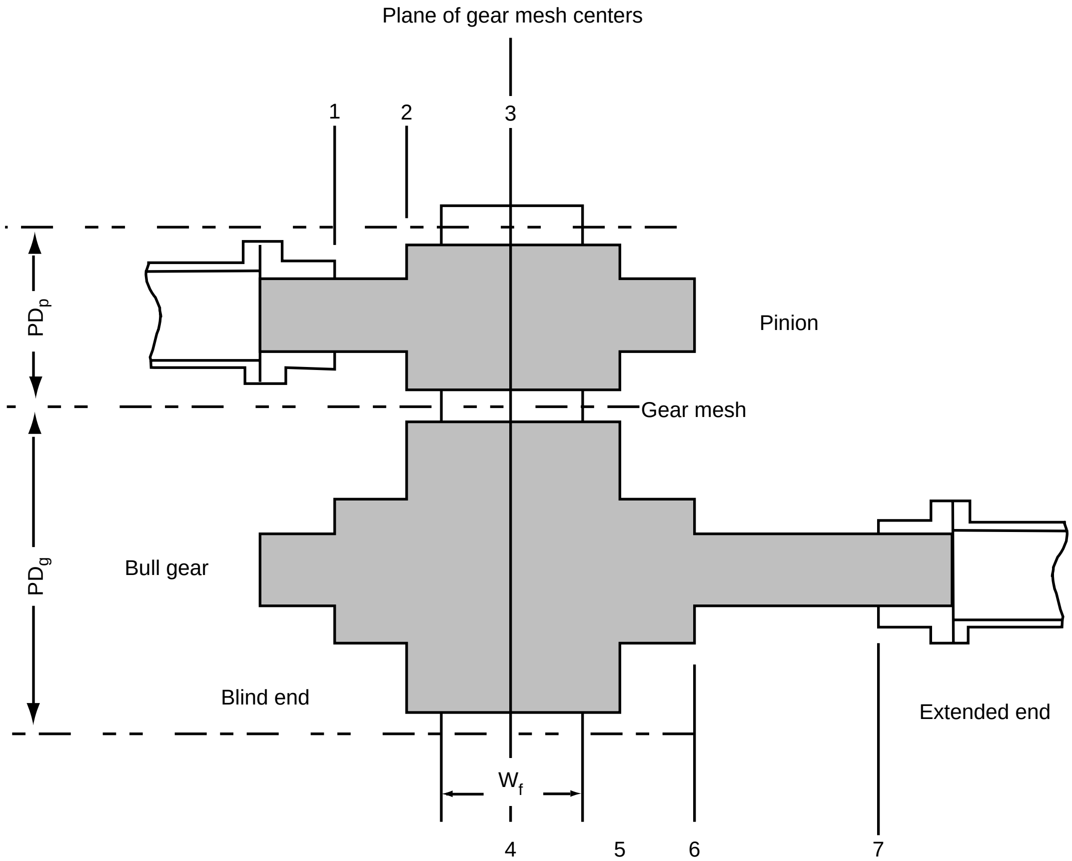
Checks on several shapes that have known exact solutions show that this equation is fairly accurate (see Table 2-2).

Table 2-2—Comparison of Exact and Approximate Results for Torsional Rigidity (Simple Geometric Cross-Sectional Shapes)

Geometric Shape	K_a/K_e
Circular Area	1.000
Square Area	1.081
Equilateral Triangle	1.140
Rectangle	1.040

In general, the preceding equation gives torsional stiffness values slightly above the exact value. This formulation can be applied to motors with various geometric cross-sections. For example, the torsional stiffness of a six-ribbed rotor whose cross-section is displayed in Figure 2-15 can be accurately estimated using the following equation:

[2]S.P. Timoshenko, *Strength of Materials*, Volume 2, Van Nostrand Company, Inc., Princeton, New Jersey, 1956.



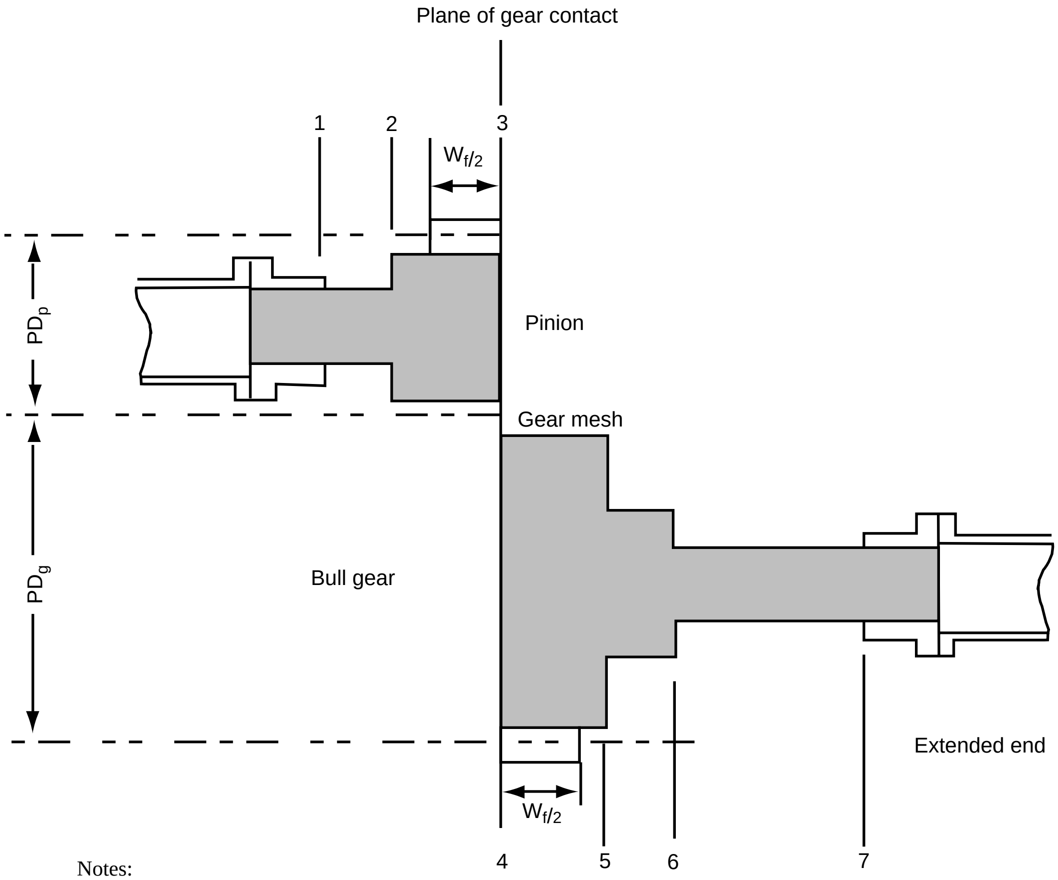
Notes:

1. PD_p = pinion pitch diameter; PD_g = gear pitch diameter; W_f = face width of mating gears. (All dimensions in mm, SI units, or inches, US Customary units.)

2. Modeling a parallel single reduction gear (see Figure 2-12 for model schematic):

—Bull gear model extends from coupling hub to center of gear mesh.
—Pinion model extends from center of gear mesh to coupling hub.
—Lump rotor inertia outboard of plane of gear contact at the center of gear mesh.
—Account for bull gear mesh penetration and stiffening.

Figure 2-13—Cross-Sectional View of a Parallel Shaft Single Reduction Gear Set



Notes:

1. PD_p = pinion pitch diameter; PD_g = gear pitch diameter; W_f = face width of mating gears.
2. The polor mass moments of inertia from removed sections are lumped at the intersection of gear shaft centerlines and the plane of gear mesh centers.

Figure 2-14—Torsional Model of a Parallel Shaft Single Reduction Gear Set

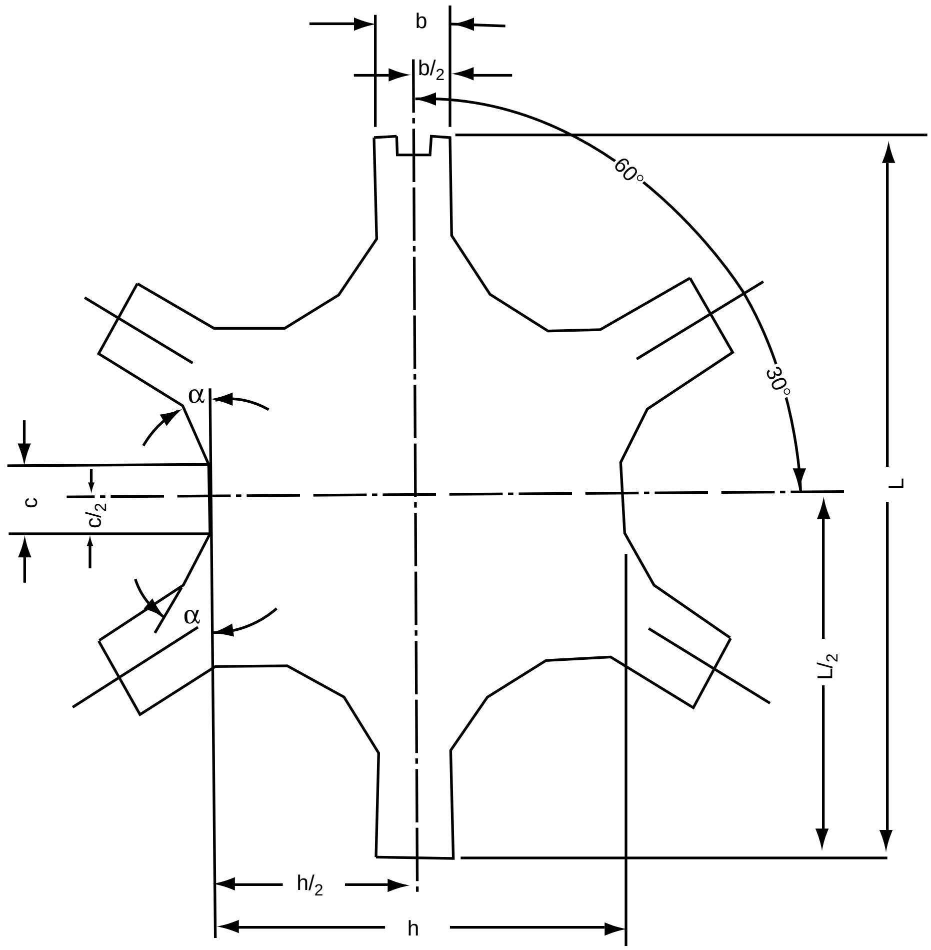
$$\frac{K_a}{K_b} = \frac{\lambda\left[1+\frac{24}{\pi}\frac{TL}{D_b^2}\right]}{1+\frac{16}{\pi}\frac{TL^3}{D_b^4}\left[1+\left(\frac{T}{L}\right)^2+3\left(1+\frac{D_b}{L}\right)^2\right]} \tag{2-8}$$

Note that

$\lambda = 1$ for integral construction.
$\lambda \leq 1$ for welded construction (typically 0.9).

Where:

Quantity	Typical SI Units	Typical US Customary Units
D_b = diameter of base shaft	meters	inches
L = arm radial length above base shaft	meters	inches
T = thickness of arm	meters	inches
K_a = equivalent stiffness of non-circular cross-section	N-m/rad	in.-lb./rad
K_b = stiffness of base shaft	N-m/rad	in.-lb./rad
λ = web construction parameter	dim	dim



Typical Units for Input

Fillet angle (α)	degrees
Rib width (b)	inches
Base length (c)	inches
Spider diameter (L)	inches
Base diameter (h)	inches

Figure 2-15—Cross Section (Perpendicular to Motor Centerline) of Motor/Generator Through the Web at Rotor Midspan

The torsional stiffness of a solid circular cylinder is written as follows:

$$K_t = \frac{G}{L}\left(\frac{\pi D^4}{32}\right) \quad (2\text{-}9)$$

Where:

Quantity	Typical SI Units	Typical US Customary Units
G = torsional modulus	N/m^2	lb/in.^{2}
L = cylinder axial length	meters	inches
D = cylinder diameter	meters	inches

From this equation, the effective diameter of a cylinder that is equal in length and torsional stiffness to the non-circular shaft section, can be calculated as follows:

$$D_{effective} = \left(\frac{32L}{\pi G}K\right)^{1/4} \quad (2\text{-}10)$$

$D_{effective}$ is the diameter of a cylinder that generates the torsional stiffness of the non-circular shaft section. The ratio of the effective cylinder diameter, $D_{effective}$, and the base shaft diameter, D_b, is written as follows:

$$\frac{D_{effective}}{D_b} = \left(\frac{K_a}{K_b}\right)^{1/4} \quad (2\text{-}11)$$

The effective diameter of the equivalent cylindrical shaft section can be calculated using this equation. Typical values of $D_{effective}$ range from 12 percent to 25 percent above the base shaft diameter, D_b, for some of the more common types of construction of multi-pole synchronous and induction machines. For further discussion see *A Handbook on Torsional Vibration*.[3]

2.6.8 ANALYSIS OF PUMPS

When formulating a torsional rotor model for a centrifugal pump with an incompressible working fluid, it becomes necessary to distinguish between dry impeller inertia and wet impeller inertia. This data in normally available from the pump manufacturer and is typically used to *band* a frequency range for the corresponding torsional natural frequencies. Although a liquid pump provides a finite level of system damping due to viscosity effects (shearing of the working fluid), a liquid pump also exhibits increased rotational inertia due to the fluid density (inertial effects of the medium).

[3]E. J. Nestorides, *A Handbook on Torsional Vibration*, B.I.C.E.R.A. Research Laboratory.

2.6.9 OTHER CONSIDERATIONS

2.6.9.1 Torsional Modulus Variation With Temperature

The torsional modulus, G, of the shaft material(s) varies with temperature. Figure 2-16 displays the effect of temperature on the shear modulus. The temperature dependence of the shear modulus should be considered in torsional analysis when large temperature changes in the shafting occurs between start-up and steady state operation. Experience indicates that temperature effects can result in a several percent difference in calculated undamped torsional natural frequencies.

2.6.9.2 Shaft Geometry Variation and Mechanical Fits

Accurate modeling of the following shaft geometry and mechanical fits (often associated with built-up shafts) is important for accurate torsional natural frequency calculation:

a. Tapered (solid or hollow) shaft sections.
b. Splined fits.
c. Curvics or serrated couplings.

2.6.9.3 Nonlinearities

An understanding is required of the varying influence of loosening fits, excessive clearances, and so forth, which cause torsional system nonlinearities that result in a loss of accuracy in the analytical predictions. Nonlinear machine elements must also be addressed and typically include the latest designs in elastomeric element and viscoelastic rotor couplings.

2.7 Presentation of Results

The primary results of the undamped torsional natural frequency analysis are the following:

a. Undamped train torsional natural frequencies and separation margins.
b. Corresponding train torsional mode shapes.

Conventional presentation of the torsional natural frequencies is made using a Campbell Natural Frequency Interference Diagram, as shown in Figures 2-17 and 2-18. Campbell diagrams provide a graphical display of a rotor system's torsional frequencies versus the frequencies of potential excitation mechanisms. The reference operating speed or speed range is also displayed. The excitation frequency lines appear as sloped lines, and represent once-per-revolution excitations on all operating shaft speeds, as well as any other significant harmonics that may be peculiar to a given system. Typical sources of steady state torsional excitation include oscillating torques in synchronous motors during startup, gear mesh run outs, 1× and 2× electrical line frequency, and any type of lateral-torsional coupling mecha-

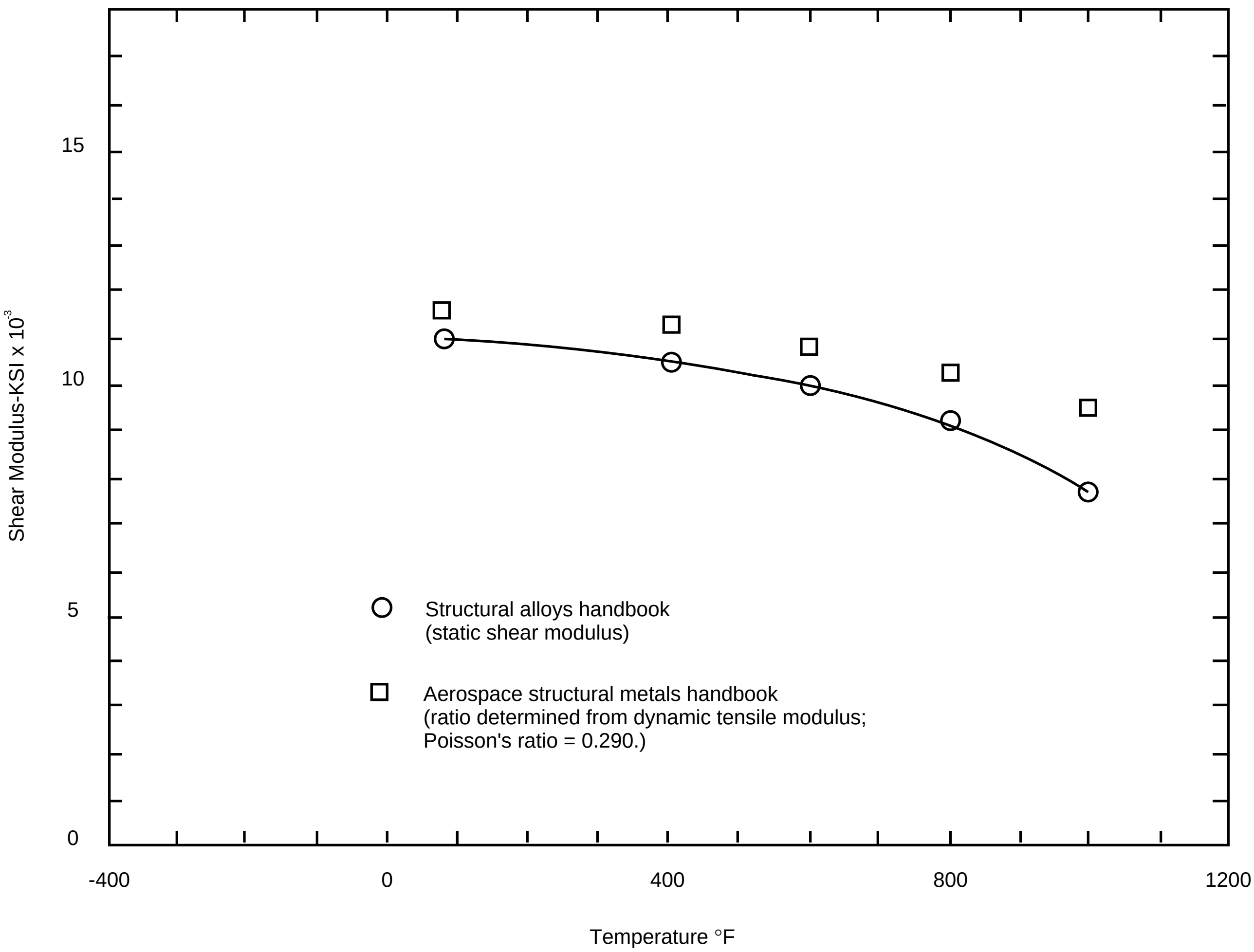


Figure 2-16— Variation of Shear Modulus With Temperature
(AISI 4140 and AISI 4340; Typical Compressor and Steam Turbine Shaft Materials)

nisms (for example, torque pulsations resulting from lateral-torsional coupling of gear vibrations). The coincidence of any torsional natural frequency with any potential excitation frequency along the reference operating speed line must meet the API separation margin of ± 10 percent.

The corresponding train rotor mode shape for each natural frequency is a plot of relative angular deflection versus axial distance along the coupled rotors. These plots are typically normalized to unity or to the location of maximum angular deflection. In Figure 2-19, Views a–e display train torsional modeshapes calculated for a motor-gear-compressor train.

Mode shape information is important to the proper interpretation of the results. Should a torsional interference exist, study of the train mode shape in question can yield information on nodal point locations and anti-nodal point locations along the deflected rotor train. This information allows the designer to understand where the system is sensitive to flexibility and where it is sensitive to inertia, respectively. This in turn allows him to affect changes in the system in order to remove the undesirable frequency from within the operating speed range.

2.8 Typical Results for Common Equipment Trains

Two sample cases are presented in order to clarify the specific results obtained from a standard torsional analysis.

2.8.1 MOTOR-GEAR-COMPRESSOR TRAIN

Figure 2-3 presents the general layout of a typical motor driven compressor train. For this example a motor, running at 1788 revolutions per minute, drives a speed increasing gear which powers an 8100 revolutions per minute compressor. This train is modeled using data normally supplied by vendors of the various components (motor, gear, compressor, and couplings) to support a torsional natural frequency anal-

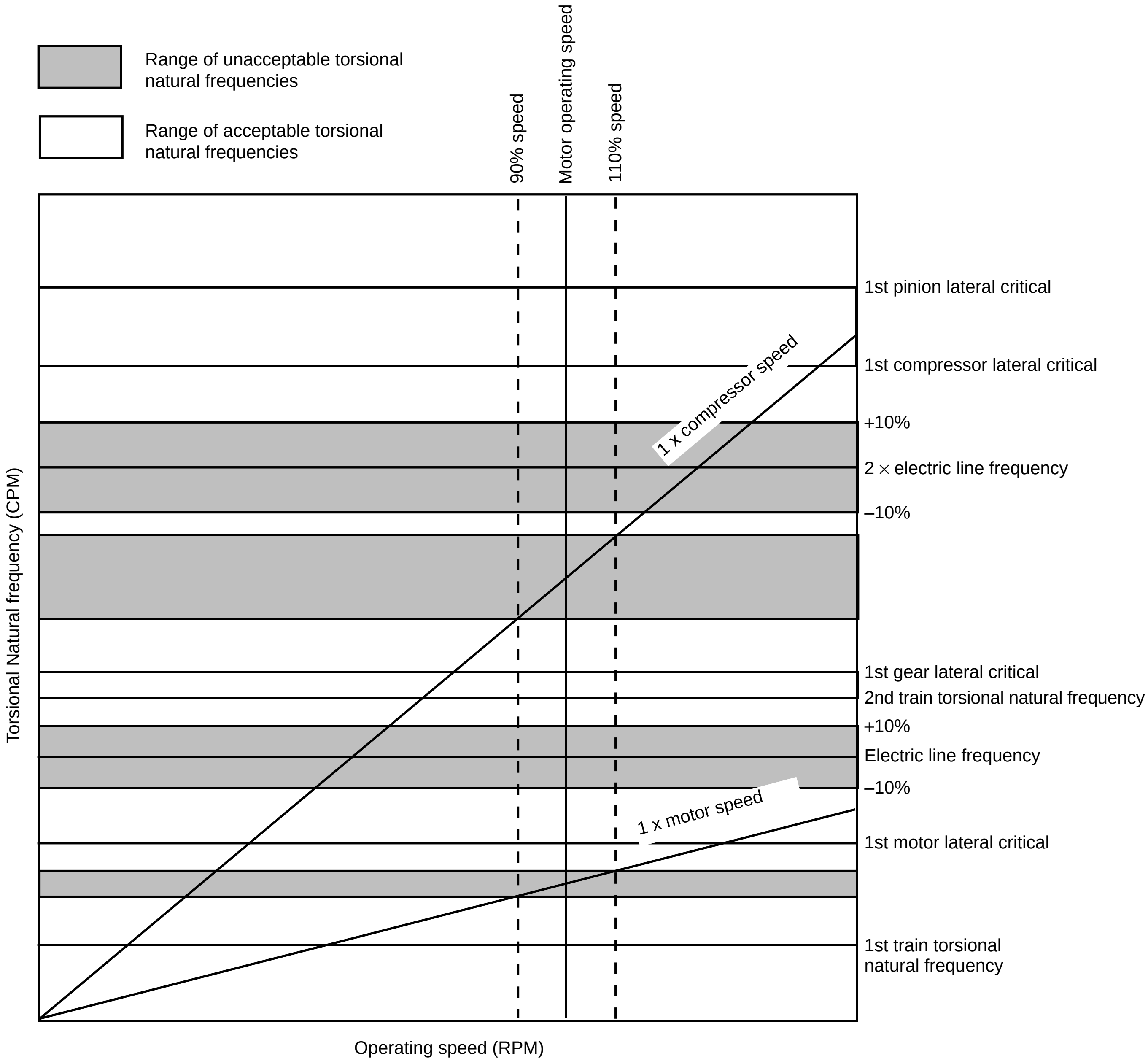


Figure 2-17—Sample Train Campbell Diagram for a Typical Motor-Gear-Compressor Train

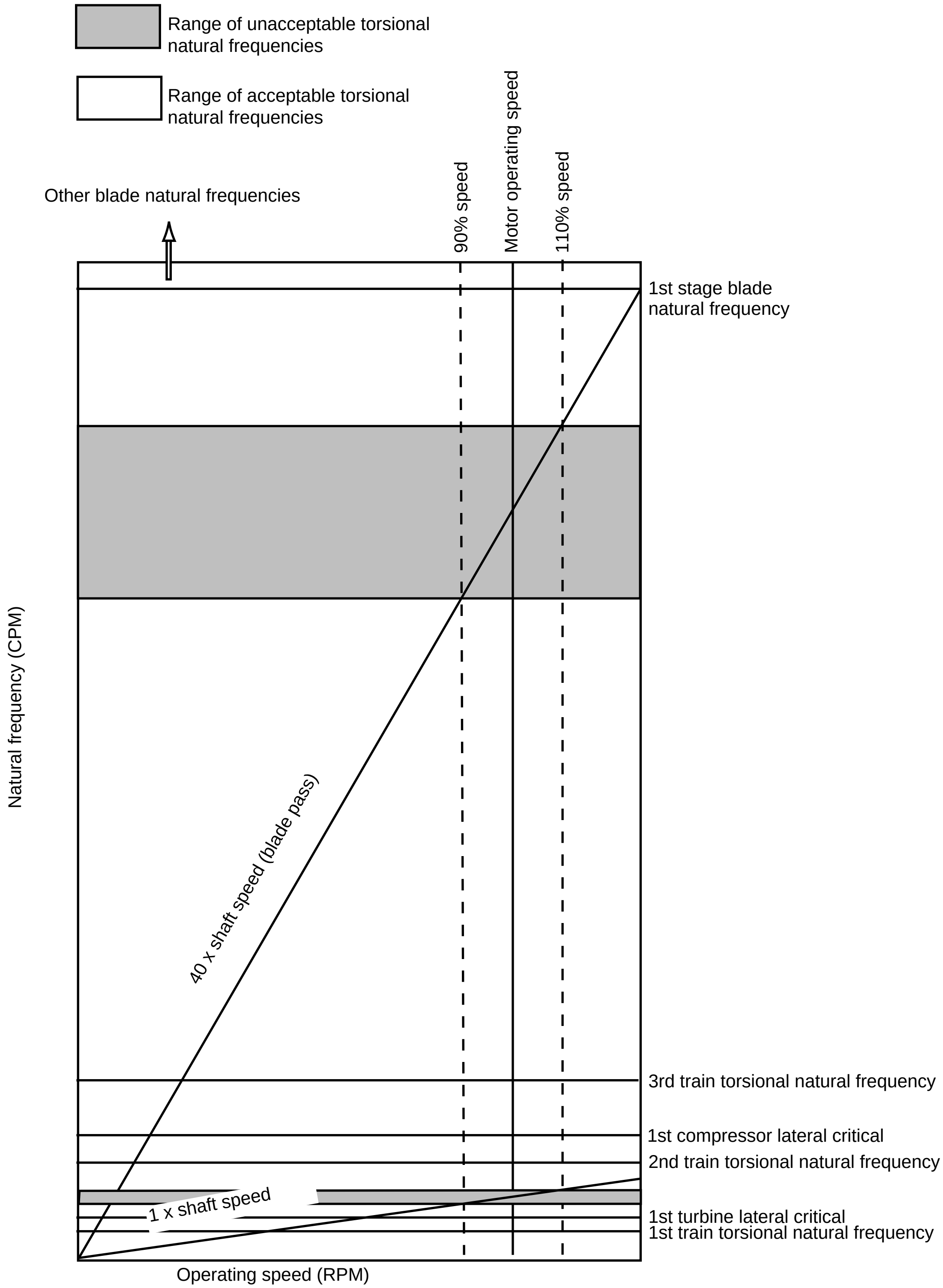


Figure 2-18—Sample Train Campbell Diagram for a Typical Turbine-Compressor Train

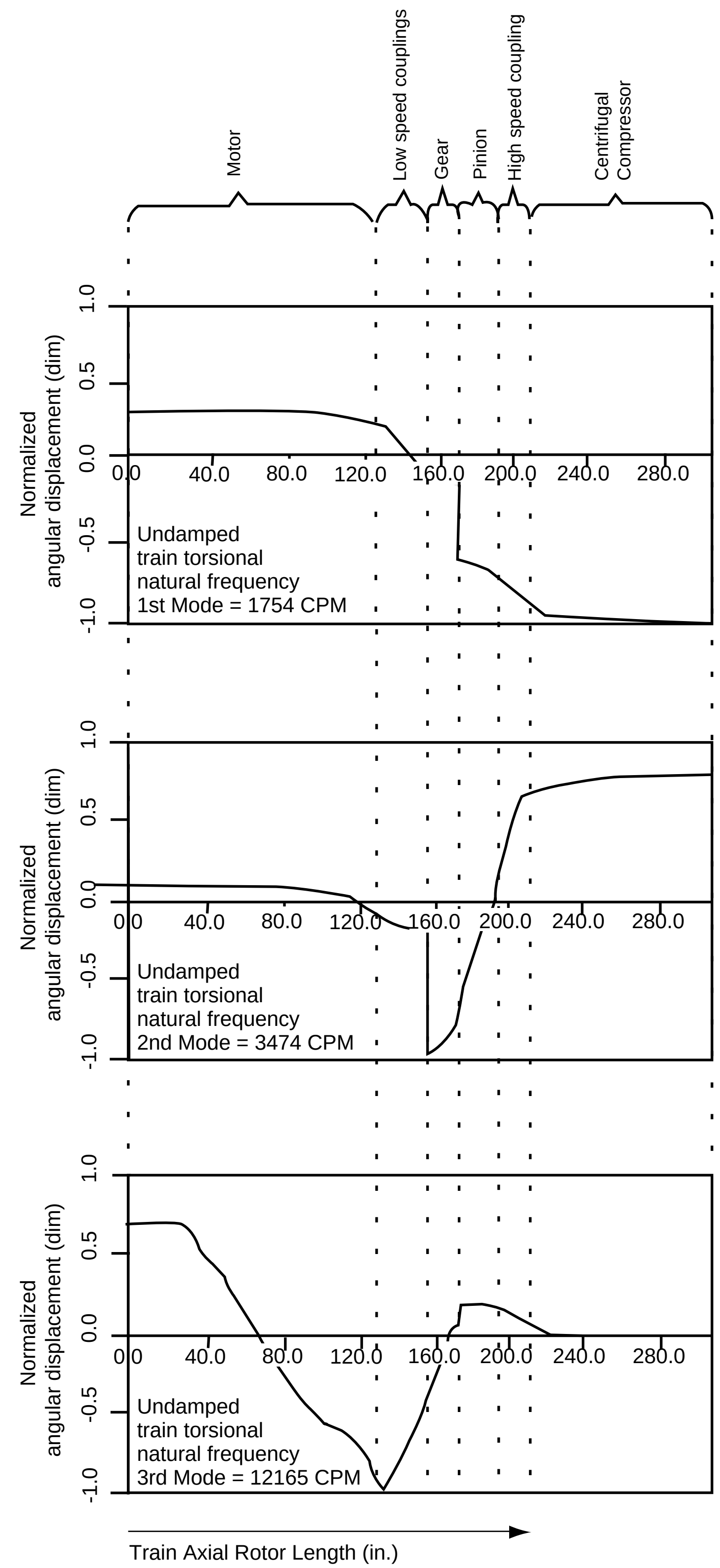


Figure 2-19—Torsional Modeshapes for a Typical Motor-Gear-Compressor Train

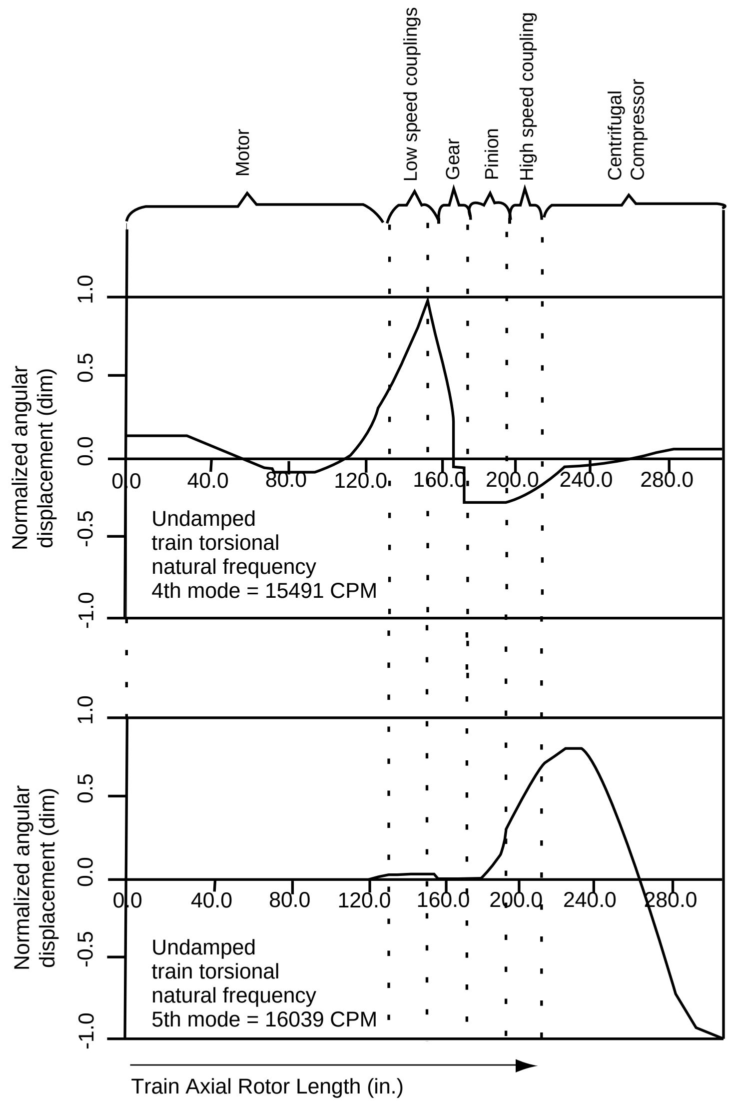


Figure 2-19—Torsional Modeshapes for a Typical Motor-Gear-Compressor Train (Continued)

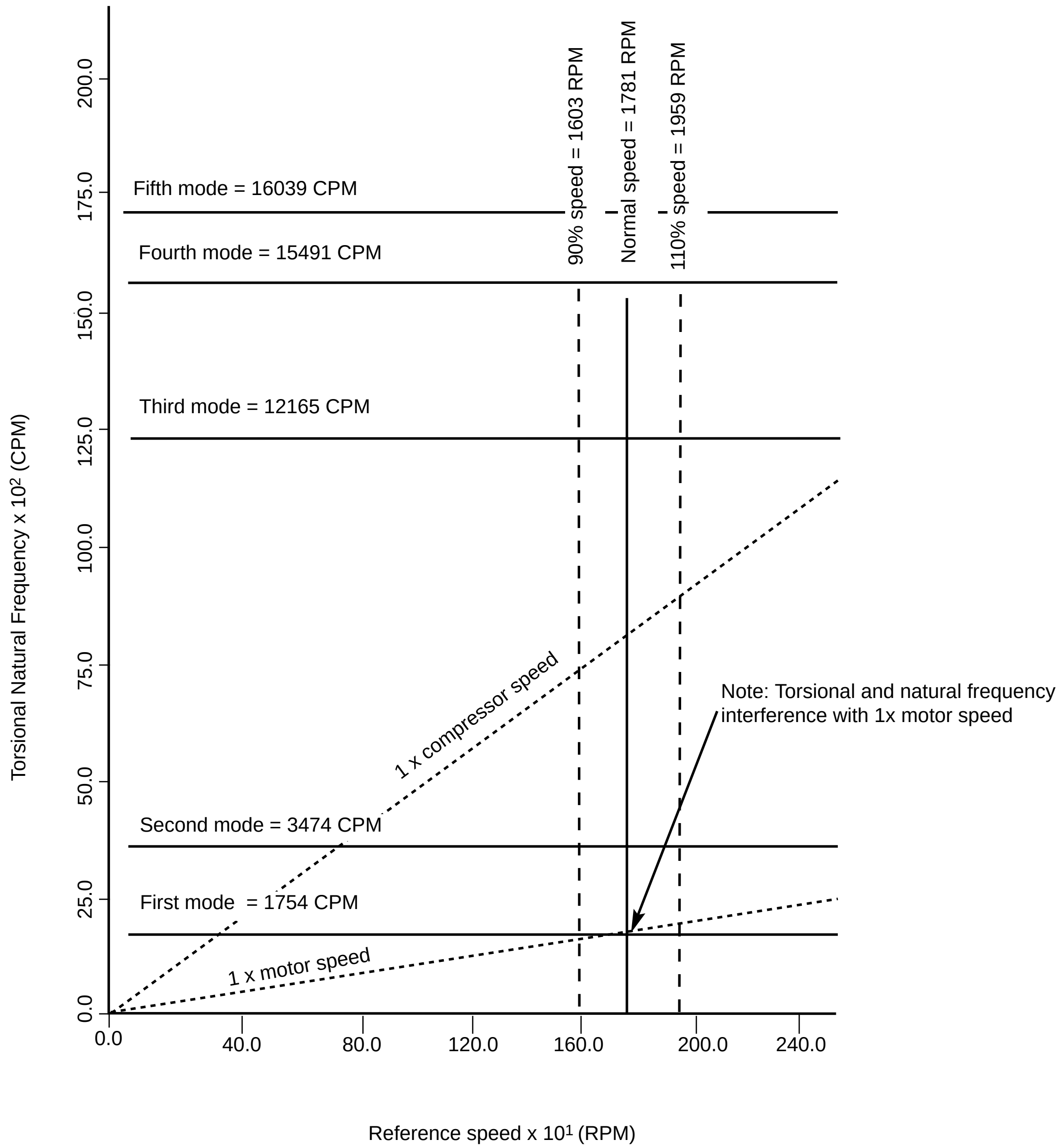


Figure 2-20—Sample Train Torsional Campbell Diagram for a Typical Motor-Gear-Compressor Train (With Unacceptable Torsional Natural Frequency Separation Margins)

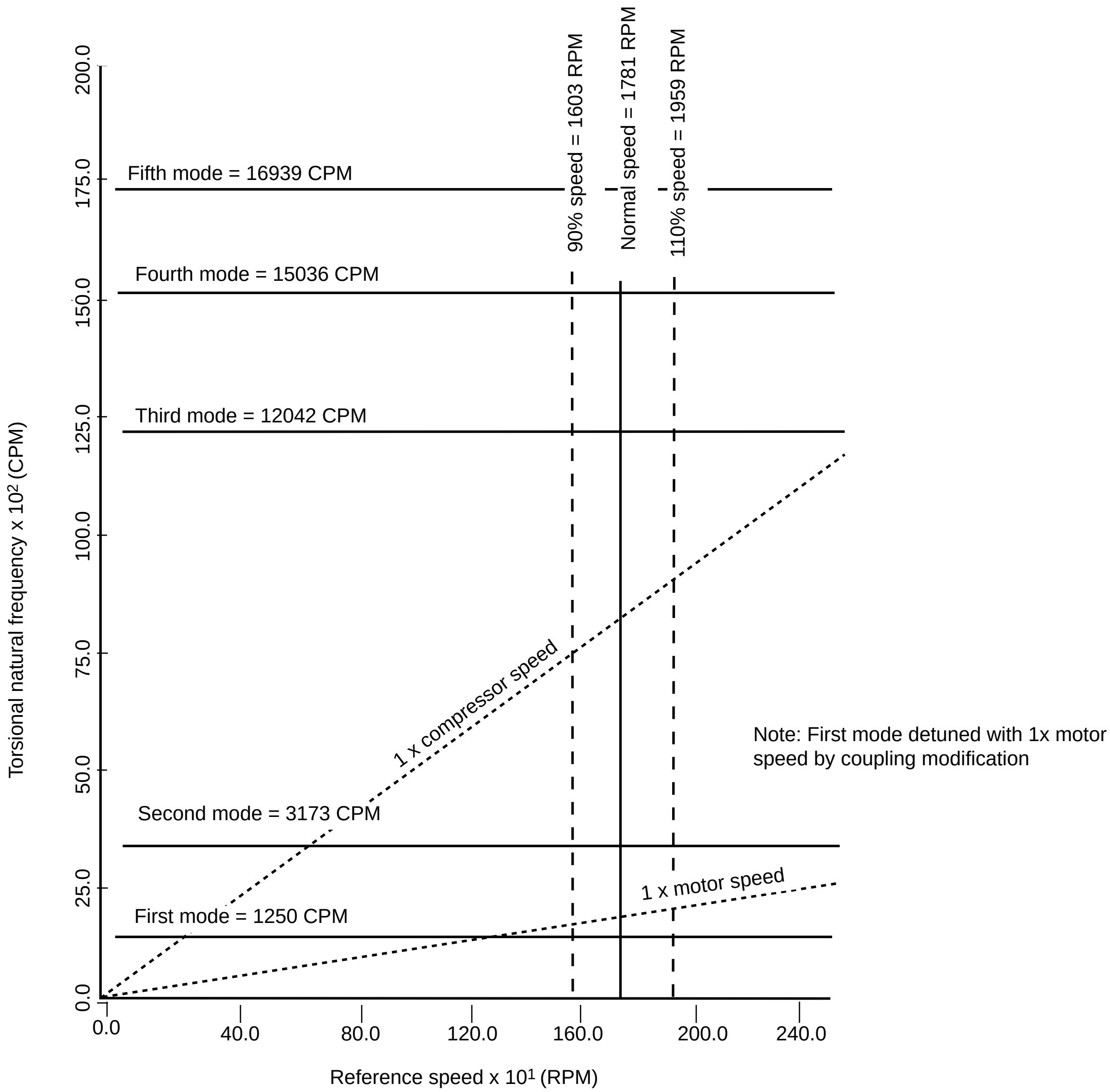


Figure 2-21—Sample Train Torsional Campbell Diagram for a Typical Motor-Gear-Compressor Train (With Acceptable Torsional Natural Frequency Separation Margins)

ysis. The Campbell diagram in Figure 2-20 cross-plots the frequencies of the modes with the shaft running speed, and Figure 2-19, Views a–e present the first five modeshapes. The Campbell diagram indicates a potential interference, within 10 percent, between the fundamental torsional mode and 1× speed. The plot also indicates that no interference exists between the 1× and 2× electrical line frequencies and any undamped torsional natural frequency. Examination of the corresponding torsional modeshape indicates that shaft twisting of the three individual units is minimal; torsional twisting is principally confined to the couplings for this mode. This implies that the torsional stiffness of the couplings is significantly lower than the torsional stiffnesses of the surrounding shafts. Hence, the frequency of this mode is governed by the torsional stiffness of the couplings. Altering the torsional stiffness of one or both couplings will allow the design engineer to shift the frequency of the potentially problematic mode. Figure 2-21 displays the result of *tuning* the coupling torsional stiffnesses so the potentially problematic mode has been shifted clear of the operating speed of the unit. Although motor drive trains typically possess more sources of torsional excitation than a turbine driven train, they are also usually limited to a single operating speed so torsional detuning is often not difficult to accomplish. Most coupling vendors will readily adjust the torsional stiffness of a coupling within a range of at least ± 25 percent. Note, however, that tuning the coupling stiffness may adversely impact the service factor of the coupling.

Careful examination of the first and second torsional modes for the motor-driven-compressor train indicates that most of the twisting occurs in the vicinity of the couplings. As just mentioned, this situation implies that the couplings are the torsionally soft elements in the train and that their torsional stiffnesses will govern the locations of the fundamental two modes. In general, machinery trains will have the same number of coupling controlled modes as couplings. These modes are torsionally significant and require de-tuning if they interfere with potential excitation frequencies. In motor-gear-compressor trains, the third mode is almost always associated with the torsional characteristics of the motor. The third mode calculated for this example is typical of motor controlled modes: calculated angular deflections are predominantly found in the low-speed shafting with the largest change in angular deflection occurring through the motor. Note that the node point of the motion is located nearly at motor midspan so that the two ends of the core vibrate out of phase. This motion is analogous to the out-of-phase free vibration observed in a system composed of two masses connected by a single spring. Higher order modes contain single unit out-of-phase motions similar to the motor controlled mode. In Figure 2-19, View e displays the compressor controlled torsional mode.

2.8.2 TURBINE-COMPRESSOR TRAIN

This example considers a steam turbine directly driving a centrifugal compressor (see Figure 2-6). A torsional analysis is usually not required for this train because the torque characteristics of the turbine provides a smooth driver with low amplitude torque pulsations in the frequency range likely to excite a lower torsional natural frequency. Without a major excitation mechanism, torsional natural frequencies will not be significantly amplified. Even in this case, however, a conservative design approach will ensure that there are no interferences with the 1× operating speed lines, particularly with the fundamental (first) torsional natural frequency.

In Figures 2-22 and 2-23, Views a–c present the train Campbell diagram and first three modeshapes for the turbine-compressor train. The Campbell diagram shows no interferences between the undamped torsional natural frequencies and the 1× operating speed lines, indicating an acceptable design for this train. Note that the coupling stiffness controlled mode lies well below the operating speed range, while the resonant modes corresponding to the particular machines lie above the operating speed range. This is characteristic of most turbine-compressor trains and results in the typically acceptable torsional characteristics for these trains.

2.9 Damped Torsional Response and Vibratory Stress Analysis

If the undamped torsional natural frequency analysis indicates an interference between an undamped torsional natural frequency and a shaft rotative speed or other potential excitation mechanism, and the train design cannot be altered sufficiently to remove the resonant interference, then a damped torsional response and vibratory stress analysis must be performed to ensure that rotor shafts and couplings are not overstressed. Potential areas of vibratory stress concentrations are couplings and shaft ends. Results generated from this type of analysis may indicate that shaft ends must be re-sized to safely accommodate the high levels of vibratory stress resulting from close operation to a torsional natural frequency. Figure 2-24 displays a typical plot of calculated oscillatory stresses versus the reference frequency (low speed shaft). The two peaks present in this plot indicate excitation of the first (1st) and second (2nd) train torsional natural frequencies by 1× and 2× operating speed torque pulsations, respectively.

Calculated stresses will be governed by assumptions regarding the level of available torsional damping as well as overall expected torque excitation levels at given frequencies. These key parameters are normally set by mutual consent of both the purchaser and the vendor and are based on experience, measurement, and/or available literature.

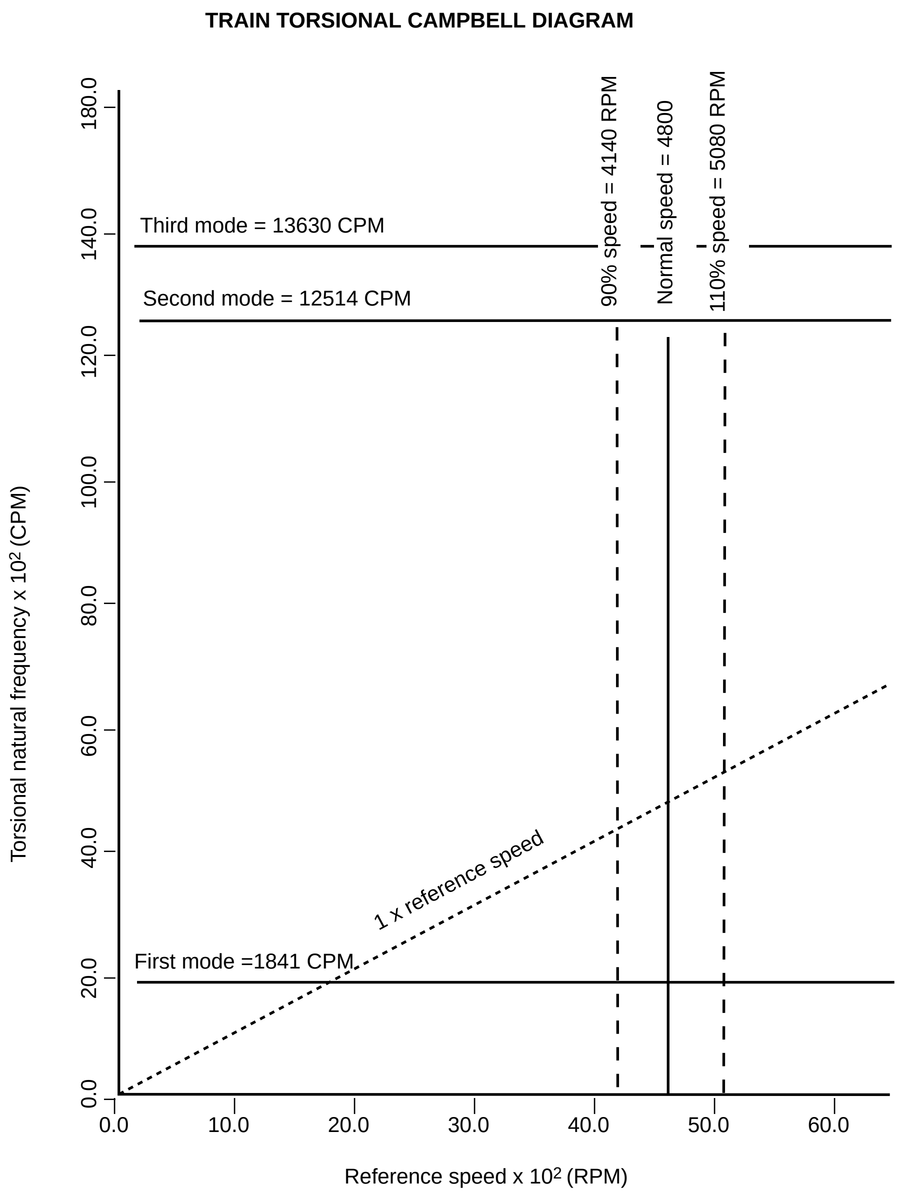


Figure 2-22—Sample Train Torsional Campbell Diagram for a Typical Turbine-Compressor Train

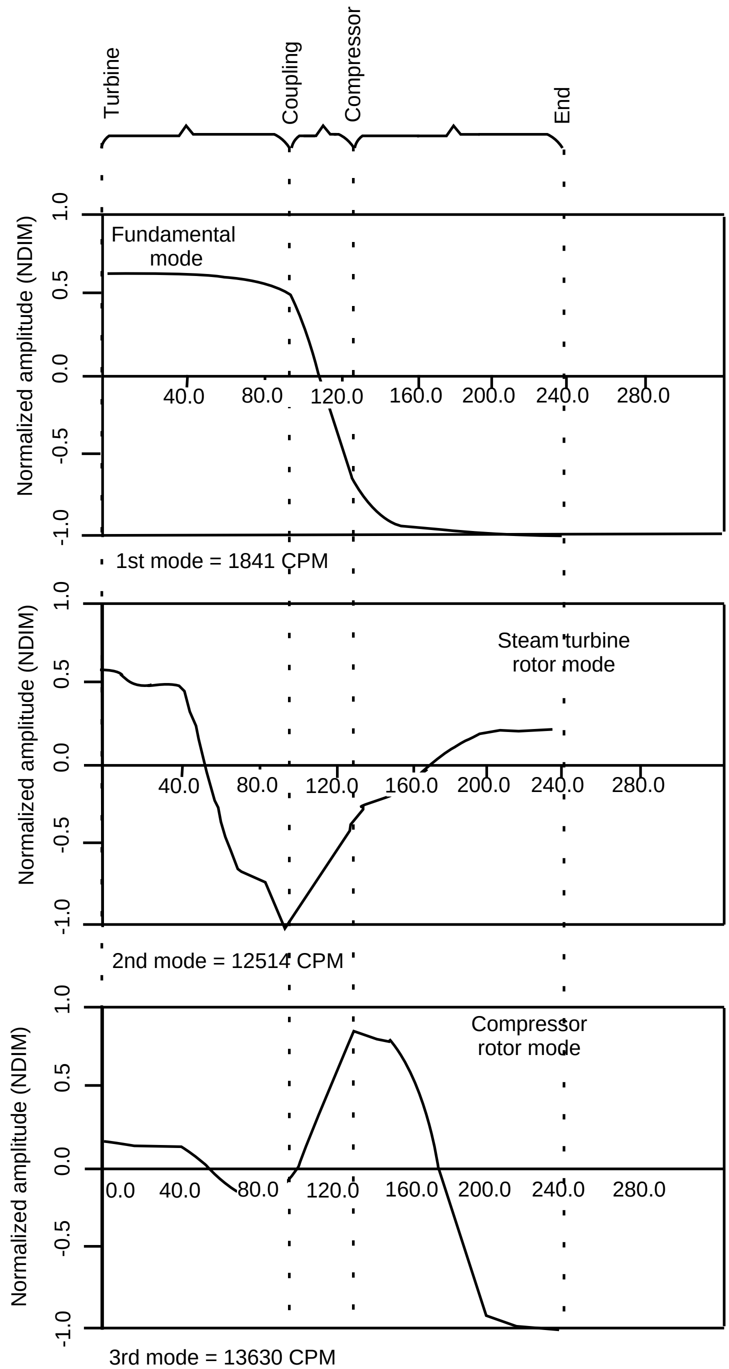


Figure 2-23—Torsional Modeshapes for a Typical Turbine-Compressor Train

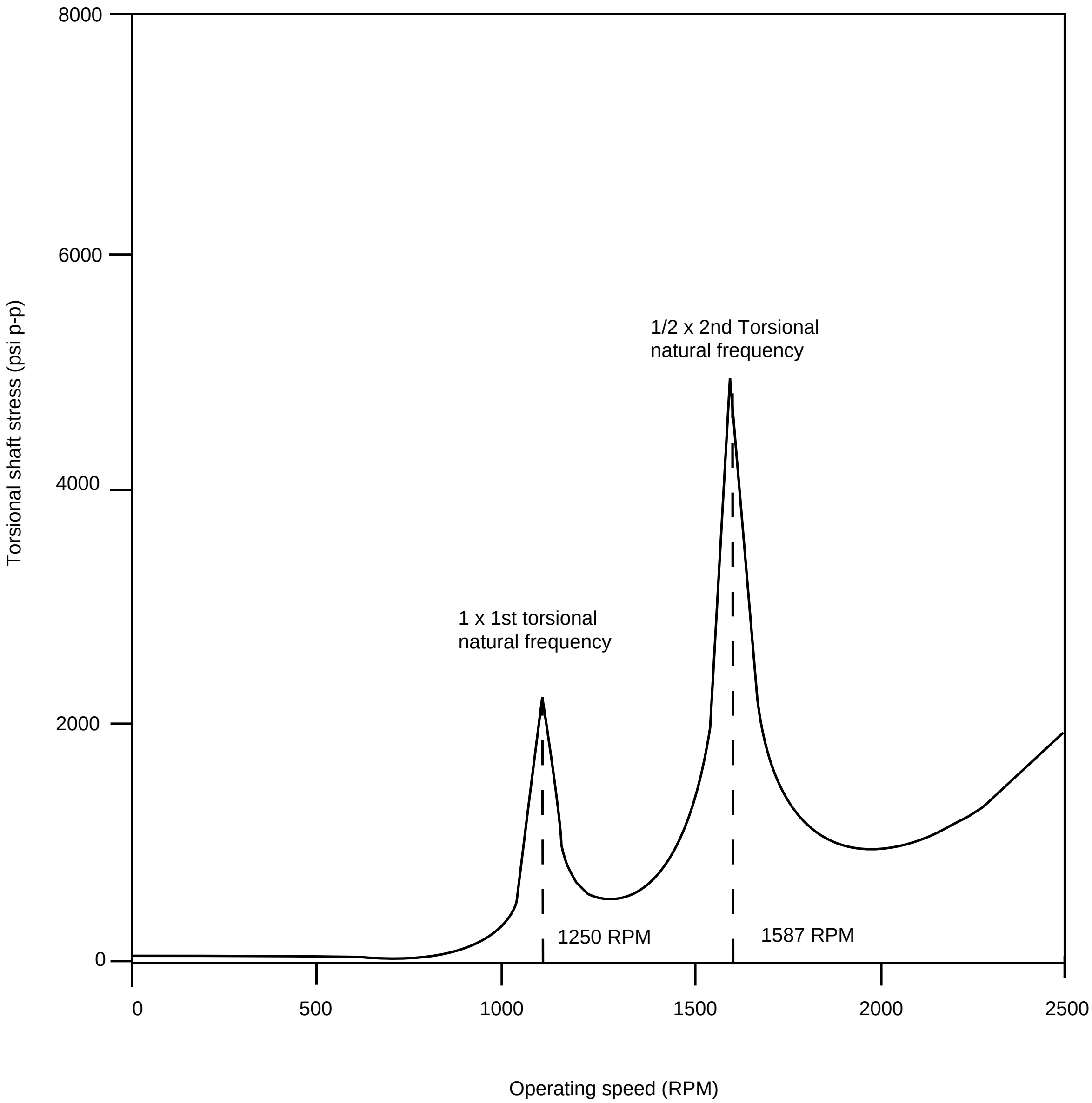


Figure 2-24—A Typical Plot of Calculated Oscillatory Stresses Versus the Reference Frequency (Low Shaft Speed)

2.10 Transient Response Analysis

Discussion in the previous section relates to the steady state torsional analysis. In some instances, it becomes necessary to analyze the time-transient characteristics of the system torsional response, requiring a more elaborate computer code. A transient torsional analysis is generally accomplished with a train model that is a reduced or condensed version of the model used to calculate the torsional undamped natural frequencies. The reduced model of the train is used to minimize the computer time required to perform the numerical solution of the torsional equations of motion during unit start-up.

Transient analysis are often required for the following cases:

a. A synchronous motor/generator that undergoes an asynchronous start-up.
b. A synchronous or induction motor that experiences a short-circuit transient fault condition or a synchronizing and/or switching transient (single/multiple reclosure).
c. A variable speed motor drive.

Variable speed motor drives require a transient torsional analysis because such motor designs result in high-level transient torque pulsations at the beginning of the unit start.

Machinery trains with synchronous motor drivers that undergo asynchronous unit starts require a transient torsional analysis to determine train response during unit start to the transient torque pulsations resulting from the oscillating air gap torque of the motor. This time-transient analysis, for the full period of train acceleration to normal running speed (synchronizing speed), is normally calculated for both full and reduced synchronous motor terminal voltage. Figure 2-25 presents a typical plot from a transient torsional analysis.

Since the pulsating component of synchronous motor torque changes linearly in frequency from 2× electric line frequency at 0 percent speed to 0 Hertz at 100 percent speed, and achieves a maximum amplitude at approximately

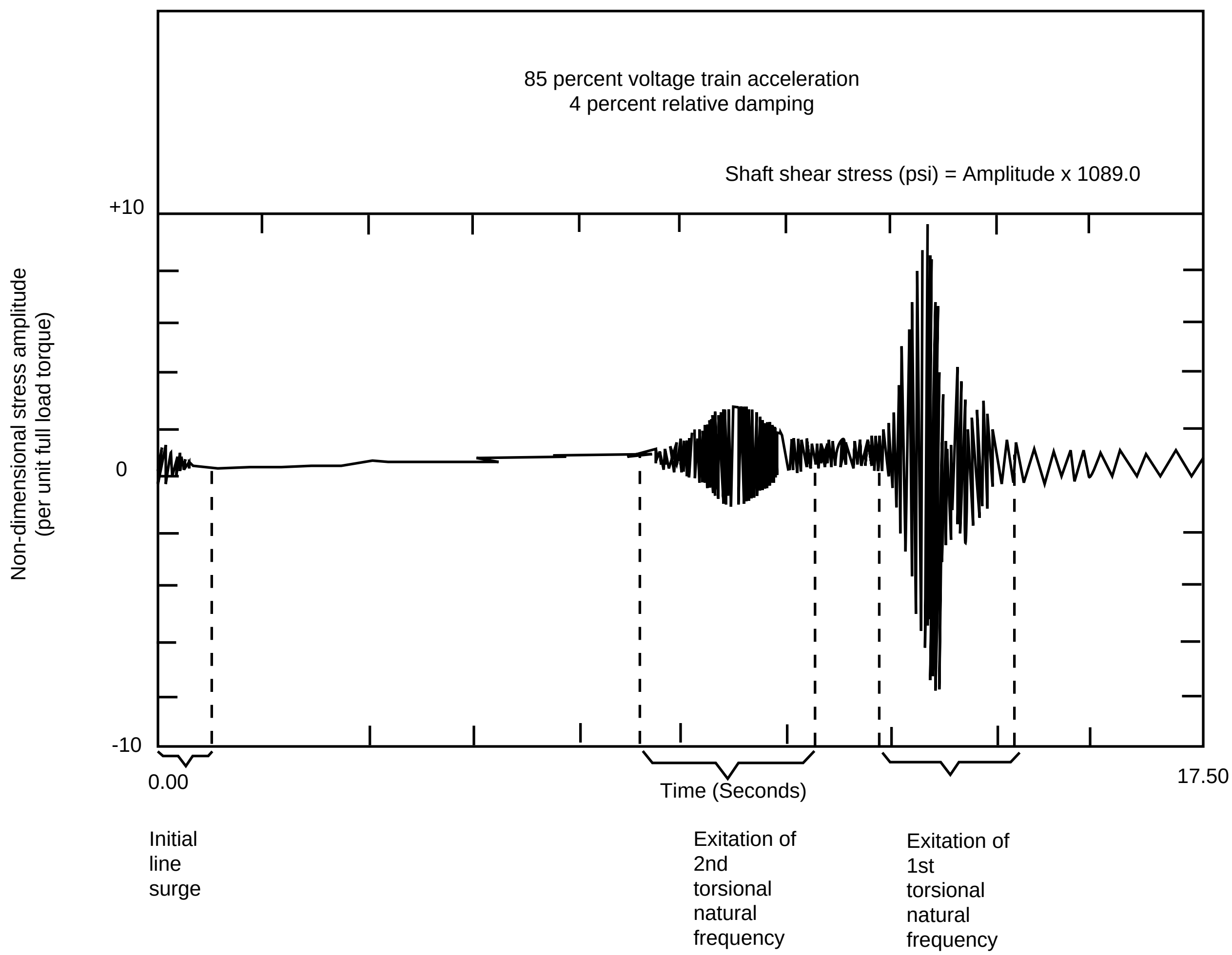


Figure 2-25—Transient Torsional Start-Up Analysis Maximum Stress Between Gear and Compressor

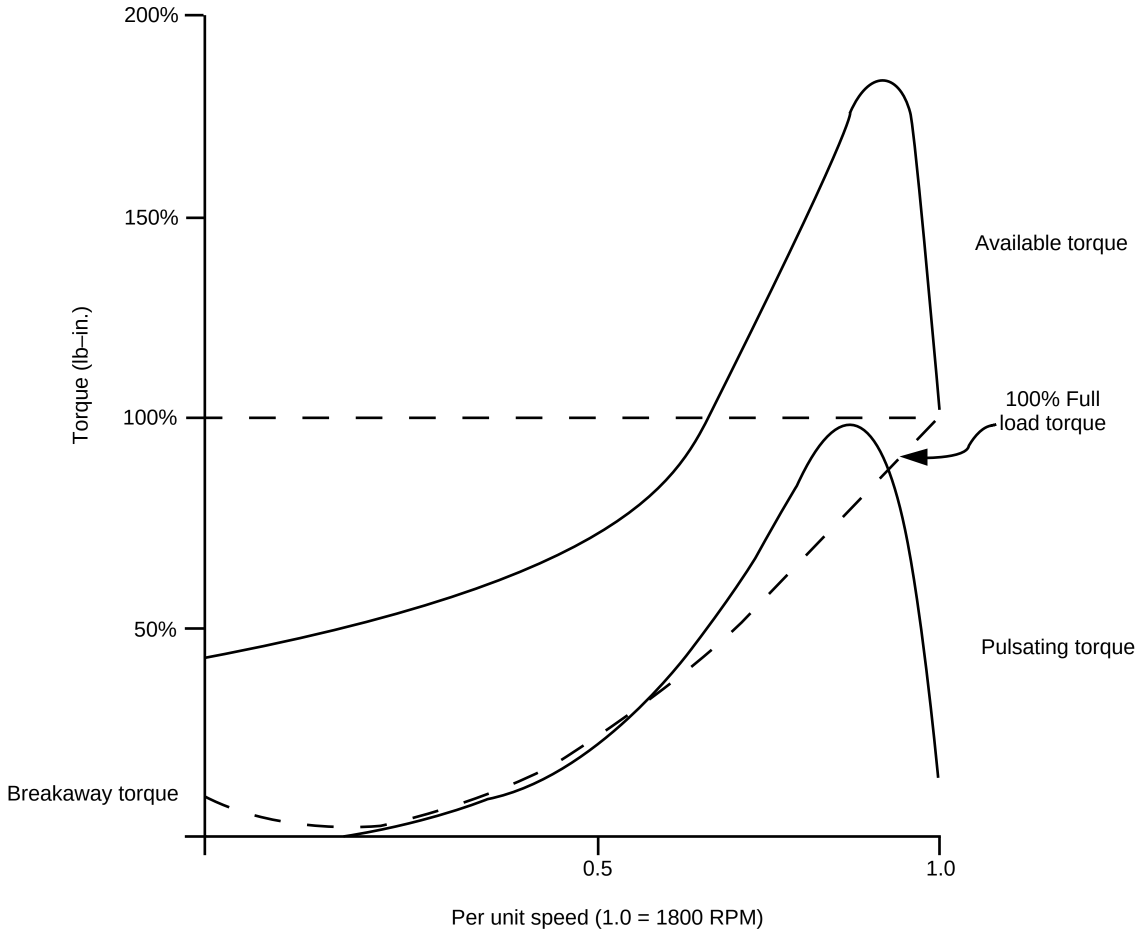


Figure 2-26—Torque Characteristics of a Typical Laminated Pole Synchronous Motor-Gear-Compressor Train

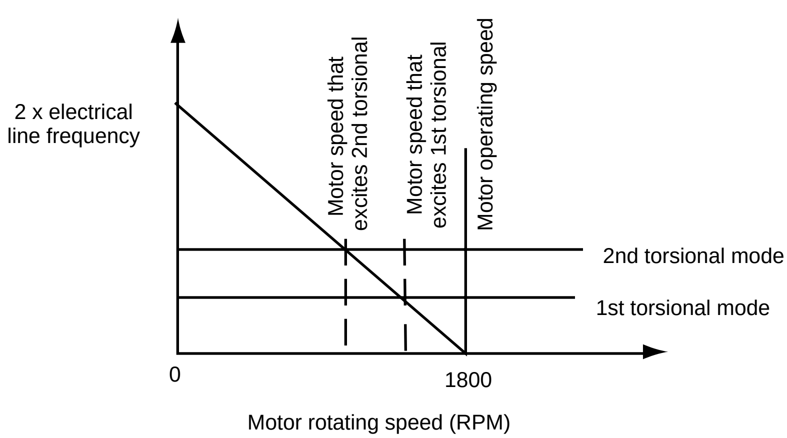


Figure 2-27—Frequency of Synchronous Motor Pulsating Torque (4 Pole Synchronous Motor)

95 percent speed (Figures 2-26 and 2-27), all system torsional natural frequencies will be excited below 7200 cycles per minute (CPM) for 60-Hertz AC or 6000 CPM for 50-Hertz systems. Typically, amplification of the second and higher modes is minimal and not of any great concern. The primary concern is the degree of amplification of the motor pulsating torque component upon traversal of the system fundamental (first) torsional mode. Preferably, judicious component selection early in a design project places the fundamental torsional mode sufficiently removed from the point of peak motor pulsation torque in order to minimize torque amplification at that frequency. When transient vibration characteristics are problematic, special couplings may be used to lower the transmission of transient torque pulsations, such as elastomeric couplings and other damper or isolator couplings.

A representative speed-torque curve for a solid pole motor is presented in Figure 2-28.

The principal results of the transient torsional analysis are given in terms of the following:

a. Maximum vibratory torque response in shafting.
b. Maximum alternating shear stress in shafting.

Maximum oscillating torque for a synchronous motor can be as high as 10–15 times full load torque for a unit. Also, when a train torsional natural frequency is traversed during startup the maximum oscillating torque encountered by shaft ends can be as high as 5–10 times the full load torque. Such levels of torque may mandate the use of larger couplings than would otherwise be required.

2.11 Design Process for Torsional Dynamic Characteristics

Use of the torsional transient analysis is part of the larger design process that results in specification of shaft-end geometries and coupling properties. Once the train has been modeled and significant torsional parameters (coupling, shaft-end properties) have been identified, the first phase of the design process is calculation of the undamped torsional natural frequencies. If adequate separation margins (±10 percent) exist between the calculated undamped torsional natural frequencies and potential torsional excitation mechanisms, then a torsional transient or other supplemental analysis is required only for special synchronous motor start-up cases, variable frequency synchronous and induction motor drives, or situations where electrical fault conditions are of concern. If the maximum oscillatory stresses in the equipment train (normally located at points where the shafts enter the coupling hubs) exceed allowable limits based on the ultimate tensile strength (UTS) of the shaft material, then either the couplings must be redesigned to lower the transient stress or the overloaded shaft end must be redesigned. If these options are not available to the vendor or user, then a fatigue analysis of the problematic shaft end must be conducted to determine the number of unit starts before shaft failure occurs.

In cases where the number of allowable unit starts is limited, the maximum oscillatory stress may exceed the endurance limit of the material. If limiting the number of unit starts is unacceptable, or if the calculated oscillatory stresses exceed the elastic limit of the shaft, then an elastomeric coupling may be applied to the train to shift the resonance frequency away from potential excitation mechanisms (for example, shaft rotation speed and motor pulsation frequencies) and to provide torsional damping to the system so the torsional responses and corresponding stresses will be reduced.

Elastomeric couplings are most often applied in cases where the fundamental mode interferes with the operating speed of the low-speed shaft. In general, elastomeric couplings should be considered when all other design efforts to remedy a torsional vibration problem have been exhausted. Elastomeric couplings can be heavier than equivalent gear or flexible diaphragm couplings and may introduce lateral rotor dynamics problems in otherwise well-behaved units. This type of coupling is also more maintenance intensive than an equivalent gear or flexible element dry coupling and may require replacement under prolonged operation.

2.12 Fatigue Analysis

Although the calculated peak torque response levels on unit starts can be below the design limits for the train components, it is sometimes necessary to calculate the approximate life of the components relative to low-cycle fatigue life to ensure the design integrity of the installation. For this analysis, the areas of highest stress, typically the machinery shaft ends, with the associated fatigue stress concentration factors, K_f, are investigated.

The basic concept of a fatigue life calculation is that each cycle of a torque signature dissipates a finite amount of the usable life of the shaft. Therefore, by counting the number and magnitude of stress cycles occurring at each torque level, the cumulative damage of each torque signature can be measured. This method of damage assessment calculates the expected number of complete torque signatures (number of starts for the train), prior to the onset of fatigue failure.

If the calculated maximum number of train starts predicted by fatigue calculations is exceeded by an estimate for number of train starts over the expected life of the train components, then this number represents a mechanical design constraint that may require redesign of some system components.

2.13 Transient Fault Analysis

This type of transient torsional analysis calculates the complex dynamic system response of a machinery train sub-

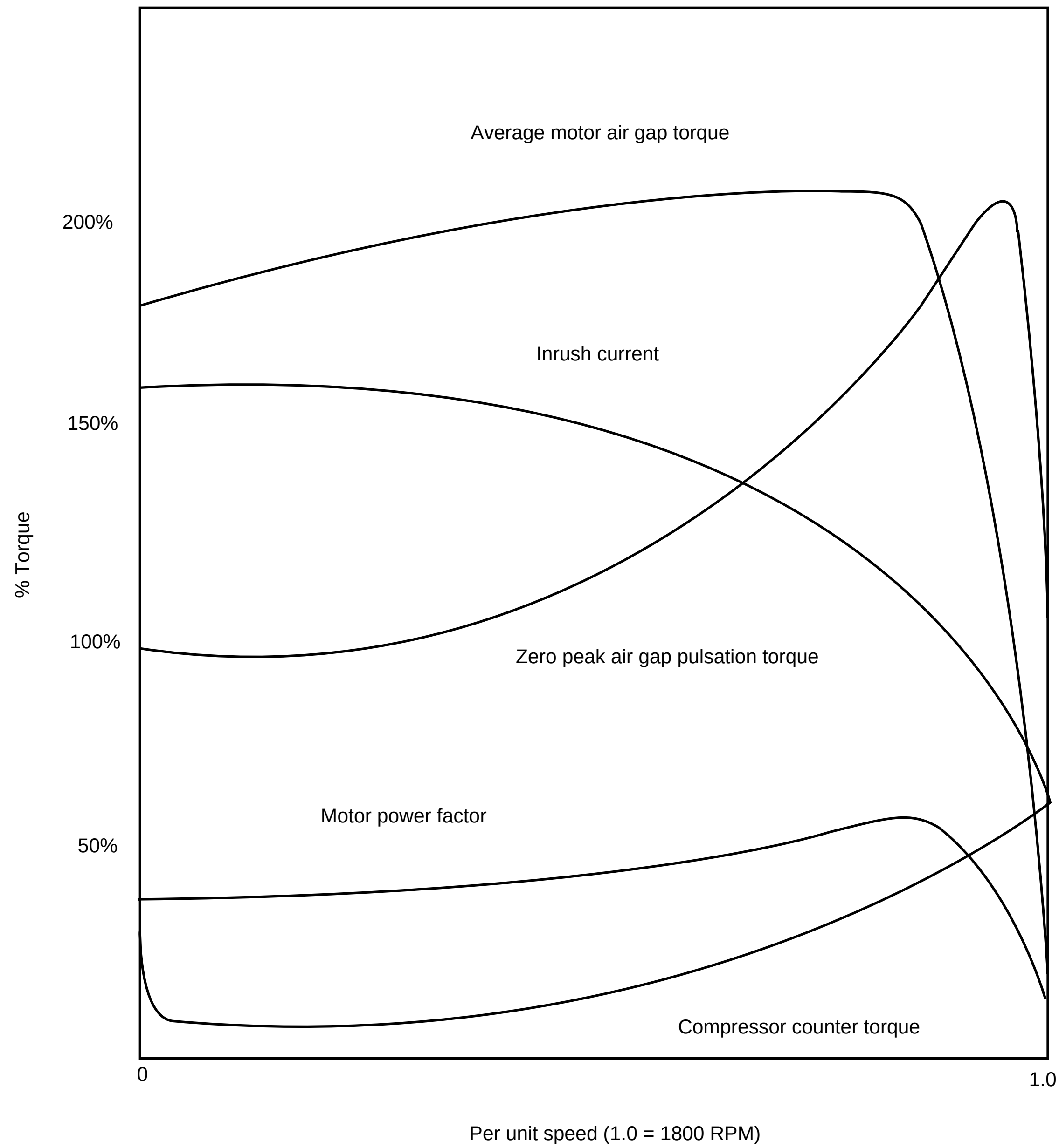


Figure 2-28—Speed-Torque Characteristics of a Solid Synchronous Pole Motor

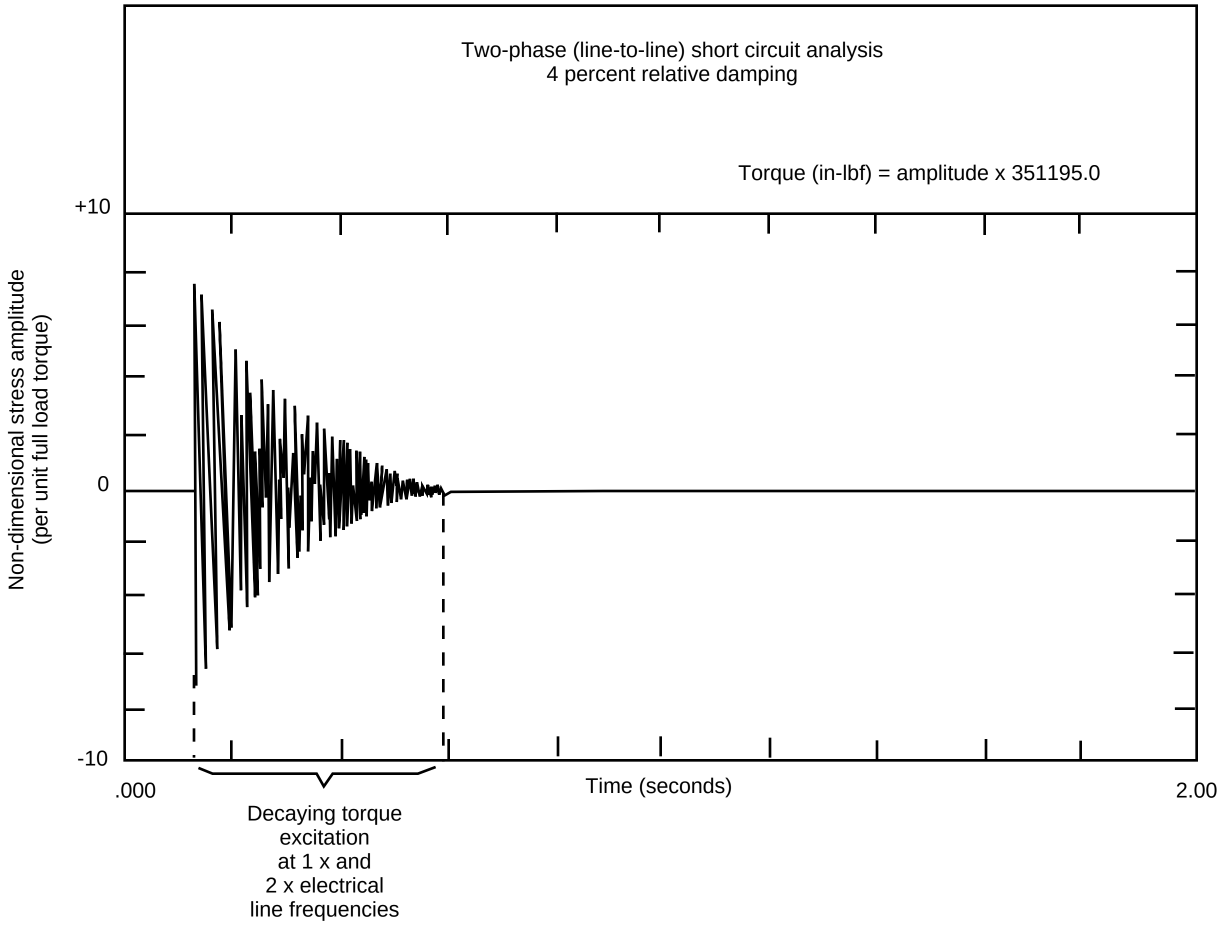


Figure 2-29—Transient Torsional Fault Analysis Worst Case Transient Fault Condition

jected to low-impedance, short circuit torque loads. The analysis is similar to a start-up transient. Figure 2-29 presents a typical plot of alternating torque resulting from a two-phase short circuit.

2.14 Testing for Torsional Natural Frequencies

The exact location of an assembled train's torsional natural frequencies may be determined during start-ups through torsional testing. Note that torsional testing is generally performed only when calculations indicate interference between a torsional natural frequency and an operating speed or some other potential torsional vibration problem. Testing for train torsional natural frequencies may be accomplished by measuring the torque transmitted through a shaft section or by measuring the absolute value of angular vibrations at a specific location. A torsiograph is an instrument that is used to measure the relative angular displacement between two points on a train. Note that the relative angular displacement between two closely spaced points on a rotating element is directly proportional to the torque transmitted by the shaft section between the two points. A torsiograph or other torque-measuring devices should be placed, when possible, in areas of the train where the oscillating torques will be amplified (that is, non-negligible) for all torsional modes of concern. The relative amplitude of the oscillating torques for each torsional mode of concern may be determined at any axial location by evaluating the slope of the undamped train torsional mode shapes.

Alternately, absolute torsional vibration displacements may be indirectly measured by attaching a ring with many equally sized, equally spaced optical or electrical (for example, notches) targets to one of the rotating elements in the train. An optical pick-up or displacement probe monitoring the target surface then generates many discrete electrical pulses during each shaft revolution. If significant torsional vibrations occur at the target ring, then the electrical pulses generated by the keyphasor probe will not be evenly spaced. Signal processing equipment converts the uneven pulse spac-

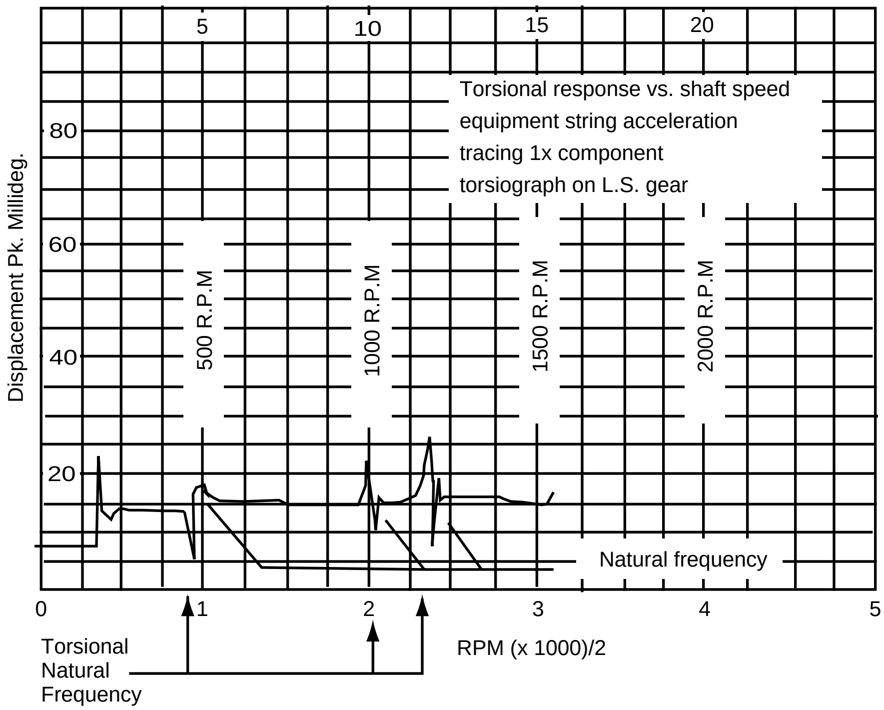


Figure 2-30—Sample Torsiograph Measurements
(Testing for Torsional Natural Frequencies)

ing into torsional (angular) displacements. The target rings should not be placed (if possible) near torsional node points for torsional modes of concern. Note that gear teeth might provide an adequate set of electrical targets for a noncontacting displacement probe as long as the gear mesh is not a node for the torsional mode(s) in question.

Figure 2-30 shows a typical plot resulting from torsiograph testing of a machinery train. This plot presents the angular displacement as a function of rotor speed, and the torsional natural frequencies are identified.

Comparison of torsional test results with analytical predictions indicates that torsional natural frequencies can be determined with a very high degree of accuracy provided that the train is accurately modeled. Typical results indicate that errors in prediction can be limited to less than 5 percent of the actual frequency.

Finally, note that motor vendors may be required to perform direct and quadrature torque testing of assembled motors in order to determine the level of torque pulsations generated by a motor. Such information may be required to either calculate or confirm maximum steady state or oscillating stress levels in train components under normal operating conditions.

APPENDIX 2A—API *STANDARD PARAGRAPHS* SECTION 2.8.4 ON TORSIONAL ANALYSIS

The following are unannotated excerpts from API *Standard Paragraphs*, 2.8.4 (R20):

2.8.4 TORSIONAL ANALYSIS

2.8.4.1 Excitations of undamped torsional natural frequencies may come from many sources, which should be considered in the analysis. These sources may include, but are not limited to, the following:

a. Gear problems such as unbalance and pitch line runout.
b. Start-up conditions such as speed detents and other torsional oscillations.
c. Torsional transients such as start-ups of synchronous electric motors and transients due to generator phase-to-phase fault or phase-to-ground fault.
d. Torsional excitation resulting from drivers such as electric motors and reciprocating engines.
e. Hydraulic governors and electronic feedback and control loop resonances from variable-frequency motors.
f. One and two times line frequency.
g. Running speed or speeds.

2.8.4.2 Undamped torsional natural frequencies of the complete train shall be at least 10 percent above or 10 percent below any possible excitation frequency within the specified operating speed range (from minimum to maximum continuous speed). [2.8.4.4]

2.8.4.3 Torsional criticals at two or more times running speeds shall preferably be avoided or, in systems in which corresponding excitation frequencies occur, shall be shown to have no adverse effect. In addition to multiples of running speeds, torsional excitations that are not a function of operating speeds or that are nonsynchronous in nature shall be considered in the torsional analysis when applicable and shall be shown to have no adverse effect. Identification of these frequencies shall be the mutual responsibility of the purchaser and the vendor.

2.8.4.4 When torsional resonances are calculated to fall within the margin specified in 2.8.4.2 (and the purchaser and the vendor have agreed that all efforts to remove the critical from within the limiting frequency range have been exhausted), a stress analysis shall be performed to demonstrate that the resonances have no adverse effect on the complete train. The acceptance criteria for this analysis shall be mutually agreed upon by the purchaser and the vendor.

● **2.8.4.5** For motor-driven units and units including gears, or when specified for turbine-driven units, the vendor shall perform a torsional vibration analysis of the complete coupled train and shall be responsible for directing the modifications necessary to meet the requirements of 2.8.4.1 through 2.8.4.4.

2.8.4.6 In addition to the torsional analysis required in 2.8.4.2 through 2.8.4.5, the vendor shall perform a transient torsional vibration analysis for synchronous-motor-driven units and/or variable speed motors. The acceptance criteria for this analysis shall be mutually agreed upon by the purchaser and the vendor.

APPENDIX 2B—TRANSFER MATRIX (HOLZER) METHOD OF CALCULATING UNDAMPED TORSIONAL NATURAL FREQUENCIES

2B.I General Description

The Transfer Matrix (Holzer) Method is best described as a method in which the output oscillating torque is calculated at one end of the train given an input oscillating torque at the other end of the train. The undamped torsional natural frequencies of the train may be calculated by noting that the magnitude of the calculated oscillating torque at the free end of the train becomes zero when the frequency of the oscillating torque matches a train natural frequency. In mathematical terms, the condition of torsional natural frequency (within a specified torque residual error) is defined as follows:

$$T_N = \sum_{i=1}^{N} I_i \omega^2 A_i = 0 \qquad \text{(2B-1)}$$

Where:

T_N = torque residual (inches-pounds).
I_i = i^{th} polar moment of inertia (pound-inches-seconds^2).
ω = frequency of oscillation (radians per second).
A_i = i^{th} shaft section coefficient.

The convergence limit for a typical transfer matrix routine is to within ± 0.01 Hertz of the actual analytical value at natural frequency. Instead of using the magnitude of the torque residual at the end of each iteration as the convergence dependent variable, most codes search for the torque residual's crossover points on the frequency axis to within the specified tolerance limit. This method is used because for all modes above the first several (which are typically controlled by the coupling torsional stiffnesses) the slope of the torque residual curve becomes very steep and may result in excessive computer iteration time if a residual torque magnitude convergence routine is employed. Since the frequencies of interest are generally several hundred CPM or larger, the error in the calculated frequency is less than ± 0.01 percent regardless of the magnitude of the torque residual.

2B.2 Limitations of Analysis

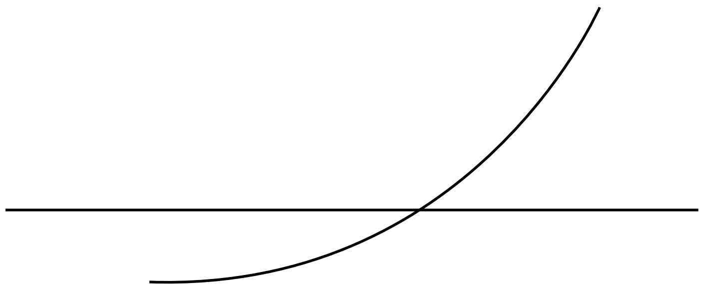
2B.2.1 SUBSYSTEM NATURAL FREQUENCY

A common occurrence in the torsional response of a rotor train is the presence of a subsystem natural frequency. This condition will yield an additional torsional natural frequency in the train analysis. However, upon closer inspection of the rotor mode shapes, it can be seen that such a frequency does not represent a true system phenomenon but, rather, is a characteristic frequency of an isolated part of the train. This subsystem natural frequency is essentially uncoupled in nature from the remainder of the system and, as such, is not a true train natural frequency. However, this frequency still represents a potentially significant vibration mode of interest in that, given the required excitation input, an undesirable resonant condition could exist. Additionally, inspection of the residual torque curve indicates that a normal cross-over point exists for a subsystem natural frequency with a finite slope at $T_N = 0$. The following are examples of such a condition:

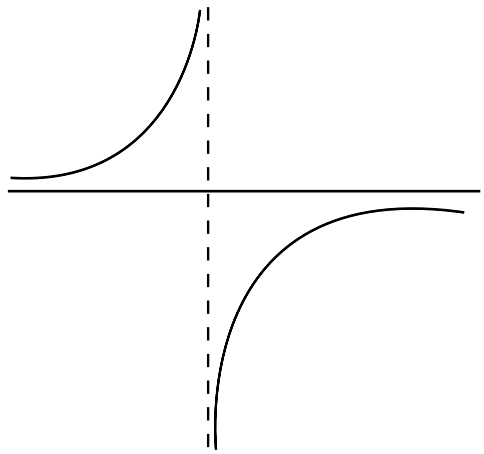
a. Exciter assemblies.
b. Multiple-geared systems, multiple branches.
c. Discontinuous systems.

2B.2.2 TUNED OSCILLATOR

Most systems yield several characteristic modes that do not represent actual resonant conditions and, as such, are of no concern to the vibrations engineer or designer. The problem lies in identifying these specific frequencies and thereby eliminating them from the remaining resonant modes of interest. These particular frequencies represent a system phenomenon whereby a part of the system is responding to the dynamics of the remainder of the system in the capacity of a tuned oscillator or vibration absorber and, therefore, does not represent a potentially resonant condition. These frequencies can be extracted by close inspection of the torque residual curve at their respective cross-over points. Tuned oscillators do not exhibit normal cross-over points in that the slope becomes infinite and the curve becomes asymptotic to the tuned oscillator frequency, with $T_N = \pm\infty$ on one side of the cross-over and $T_N = \pm\infty$ on the opposite side (Figures 2-31 a–b). Extracting these modes can be difficult and must be done with caution. It is especially significant to recognize that tuned oscillators exist and that the Holzer technique will yield their frequencies as if they were true natural frequencies. Predominance of these frequencies can be found in more complicated systems, especially multi-branch or multi-geared systems.



Normal crossover point
(a)



Tuned oscillator crossover point
(b)

Figure 2-31—Crossover Points on Torque Residual Curves

SECTION 3—INTRODUCTION TO BALANCING

3.1 Scope

Unbalance of rotating machinery parts is the most common cause of equipment vibration. The API Subcommittee on Mechanical Equipment has, therefore, developed equipment standards for allowable unbalance levels to minimize the effect of unbalance on overall equipment vibration. Precision dynamic balance of rotating machinery is required to ensure that the equipment performs with minimal vibration on both the vendor test stand and in the field.

The purpose of this tutorial is to acquaint both manufacturers and users with the API Subcommittee on Mechanical Equipment requirements for dynamic balancing, the reasoning behind these requirements, and the balancing techniques necessary to achieve these requirements.

With proper balancing, the desired end result is a machine that operates at very low levels of vibration on the test stand and in the field. The lower the level of vibration, the lower the stresses and forces acting upon the machine's rotor, bearings, and support system. This affects the overall reliability and service life of the equipment as well as providing some allowance for in-service rotor erosion, fouling, and so forth.

Appendix 3A of this tutorial gives unannotated excerpts from the API *Standard Paragraphs*, 2.8.5 on vibration and balance.

3.2 Introduction

3.2.1 DEFINITION OF TERMS

3.2.1.1 *Balance resonance speed(s)* is a shaft rotative speed(s) [or speed region(s)] which equals a natural frequency of the rotor system. When a rotor accelerates or decelerates through this speed region(s), the observed vibration characteristics are (a) a peak in the 1× vibration amplitude and (b) a change in the phase angle.

3.2.1.2 *Balanced condition* is a condition where the mass centerline (principal inertial axis) approaches or coincides with the rotor rotational axis, thus reducing the lateral vibration of the rotor and the forces on the bearings, at once per revolution frequency (1×).

3.2.1.3 *Balancing* is a procedure for adjusting the radial mass distribution of a rotor so that the mass centerline (principal inertial axis) approaches or coincides with the rotor rotational axis, thus reducing the lateral vibration of the rotor due to imbalance inertia forces and forces on the bearings, at once-per-revolution frequency (1×).

3.2.1.4 *Bow* is a shaft condition such that the geometric shaft centerline is not straight. Usually the centerline is bent in a single plane due to gravity sag, thermal warpage, and so forth; however, the bow may be three dimensional (corkscrew). Shaft bow can be detected by measuring the shaft relative displacement at slow roll speed.

3.2.1.5 *Calibration* is a test during which known values of the measured variable are applied to the transducer or readout instrument and corresponding output readings are verified or justified as necessary.

3.2.1.6 *Calibration weight* is a weight of known magnitude which is placed on the rotor at a known location in order to measure the resulting change in machine vibration (1× vector) response. In effect, such a procedure *calibrates* the rotor system (a known input is applied, and the resultant output is measured) for its susceptibility to unbalance. Calibration weight is sometimes called *trial weight*.

3.2.1.7 *Critical speed(s)* is defined in the standard paragraphs as a shaft rotational speed that corresponds to a non-critically damped ($AF > 2.5$) rotor system resonance frequency. According to API, the frequency location of the critical speed is defined as the frequency of the peak vibration response as defined by the Bodé plot resulting from a damped unbalance response analysis and shop test data.

3.2.1.8 *Eccentricity, mechanical* is the variation of the outer diameter of a shaft surface when referenced to the true geometric centerline of the shaft: out-of-roundness.

3.2.1.9 *Electrical runout* is a source of error on the output signal of a proximity probe transducer system resulting from non-uniform electrical conductivity/resistivity/permeability properties of the observed material shaft surface: a change in the proximitor output signal that does not result from a probe gap change.

3.2.1.10 *Heavy spot* is a term used to describe the position of the unbalance vector at a specified lateral location (in one plane) on a rotor.

3.2.1.11 *High spot* is the term used to describe the response of the rotor shaft due to unbalance force.

3.2.1.12 *Imbalance (unbalance)* is a measure that quantifies how much the rotor mass centerline is displaced from the centerline of rotation (geometric centerline) resulting from an unequal radial mass distribution on a rotor system. Imbalance is usually given in either gram-centimeters or ounce-inches.

3.2.1.13 *Influence vector* is the net 1× vibration response vector divided by the calibration weight vector (trial weight vector) at a particular shaft rotative speed. The measured vibration vector and the unbalance force vector represent the rotor's transfer function.

3.2.1.14 *Keyphasor* is a location on the shaft circumference which provides a once-per-revolution occurrence. The occurrence can be a keyway, a key, a hole or slot, or a projection.

3.2.1.15 *Mechanical runout* is a source of error on the output signal of a proximity probe transducer system, a probe gap change which does not result from either a shaft centerline position change or shaft dynamic motion. Common sources include out-of-round shafts, scratches, dents, rust, or other conductive buildup on the shaft, stencil marks, flat spots, and engravings.

3.2.1.16 *Microinch* is a unit of length or displacement equal to 10^{-6} inches or 10^{-3} mils. A *mil* is a unit of length or displacement equal to 0.001 inch. One mil equals 25.4 micrometers.

3.2.1.17 *Natural frequency* is synonymous with *resonant frequency*.

3.2.1.18 *Nonsymmetric rotor* is a rotor whose cross-section has two different geometric moments of inertia (for example, an elliptical cross-section), and/or the supports have different characteristics in the horizontal and vertical directions.

3.2.1.19 *Orbit* is the dynamic path of the shaft centerline displacement motion as it vibrates during shaft rotation.

3.2.1.20 *Resonance* is described by API as the manner in which a rotor vibrates when the frequency of a harmonic (periodic) forcing function coincides with a natural frequency of the rotor system. When a rotor system operates in a state of resonance, the forced vibrations resulting from a given exciting mechanism (such as unbalance) are amplified according to the level of damping present in the system. A resonance is typically identified by a substantial vibration amplitude increase and a rapid shift in phase angle.

3.2.1.21 *Synchronous* is the component of a vibration signal that has a frequency equal to the shaft rotative frequency (1×).

3.2.2 UNITS FOR EXPRESSING UNBALANCE

The amount of unbalance in a rotating assembly is normally expressed as the product of the unbalance weight (for example, ounces and grams) and its distance from the rotating centerline (such as inches and centimeters). Thus, the units for unbalance are generally ounce-inches, gram-inches, gram-centimeters, and so forth. For example, one ounce-inch of unbalance would equate to a heavy spot on a rotor of one ounce located at a radius of one inch from the rotating centerline. Figure 3-1 illustrates this example of unbalance expressed as the product of weight and distance.

3.2.3 API STANDARD BALANCE SPECIFICATIONS

Balancing denotes the attempt to improve the mass distribution of a rotating assembly such that the assembly rotates in its bearings with minimal unbalanced centrifugal forces. This goal, however, can be achieved only to a certain degree; even after careful balancing of a given rotor is completed,

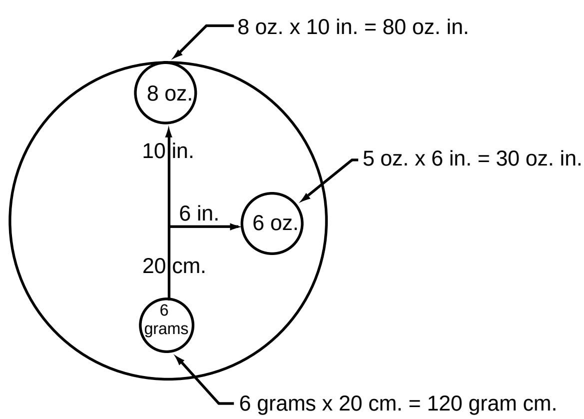


Figure 3-1—Units of Unbalance Expressed as the Product of the Unbalance Weight and its Distance From the Center of Rotation

the rotor will still retain a certain degree of unbalanced mass distribution known as *residual unbalance*.

The API Standard Paragraph's specifications for residual unbalance was originally adopted from the US Navy, Bureau of Ships Standards.

For Customary Units, use the following equation:

$$U = 4W/N = \text{Ounce-Inches} \tag{3-1}$$

For SI Units, use the following equation:

$$U = 6350W/N = \text{Gram-Millimeters} \tag{3-2}$$

Where:

U = maximum allowable residual unbalance for each correction plane. The tolerance for each plane is based on the static weight supported at each end of the rotor.

W = bearing journal static weight at each end of the rotor. W is expressed in pounds for customary units or kilograms for metric units. Note that for relatively uniform rotors, W represents ½ the total rotor weight.

N = maximum continous rotor speed in revolutions per minute (Note: Not the balance speed).

The force generated due to a rotor's residual unbalance can be shown by the following formula:

For Customary Units:

$$F_{unb} = 1.77(RPM/1000)^2 \times R_{unb} \tag{3-3}$$

Where:

F_{unb} = radial force generated (in pounds) due to the residual unbalance of the rotating assembly.

RPM = the maximum continuous speed of the rotating assembly (in revolutions per minute).

R_{unb} = the residual unbalance (in ounce-inches) of the rotating assembly.

For SI units:

$$F_{unb} = 0.01(RPM/1000)^2 \times R_{unb} \tag{3-4}$$

Where:

F_{unb} = radial force generated (in kilograms) due to the residual unbalance of the rotating assembly.

RPM = the maximum continuous speed of the rotating assembly (in revolutions per minute).

R_{unb} = the residual unbalance (in gram-centimeters) of the rotating assembly.

From the above formulas, it can be seen that the centrifugal force due to unbalance increases by the square of the rotor speed and that these forces can be significant for relatively small amounts of unbalance. For example, a rotor with a residual unbalance of only 432 gram-centimeters (6 ounce-inches) running at 10,000 RPM will generate a force due to unbalance of over 430 kilograms (1000 pounds.)

The API *Standard Paragraphs* for residual unbalance provides an achievable unbalance level that will minimize the affect of rotating unbalance on overall vibration level.

Allowable vibration level and unbalance tolerance are two separate subjects; they both, however, are the means to an end: a smooth running machine. The lower a machine's vibration level is upon commissioning, the longer the service life that machine stands to achieve.

3.2.4 BALANCING TOLERANCES

The present-day state of the art in balancing technology is such that it is not uncommon for high-speed turbomachinery rotors to operate with shaft vibration levels of 12.5 microns (0.5 mils) or less. Achieving vibration levels this low requires sound balance practices and tight residual unbalance tolerances. In establishing balance tolerances, there is always the trade-off between what is practically feasible and what is economical.

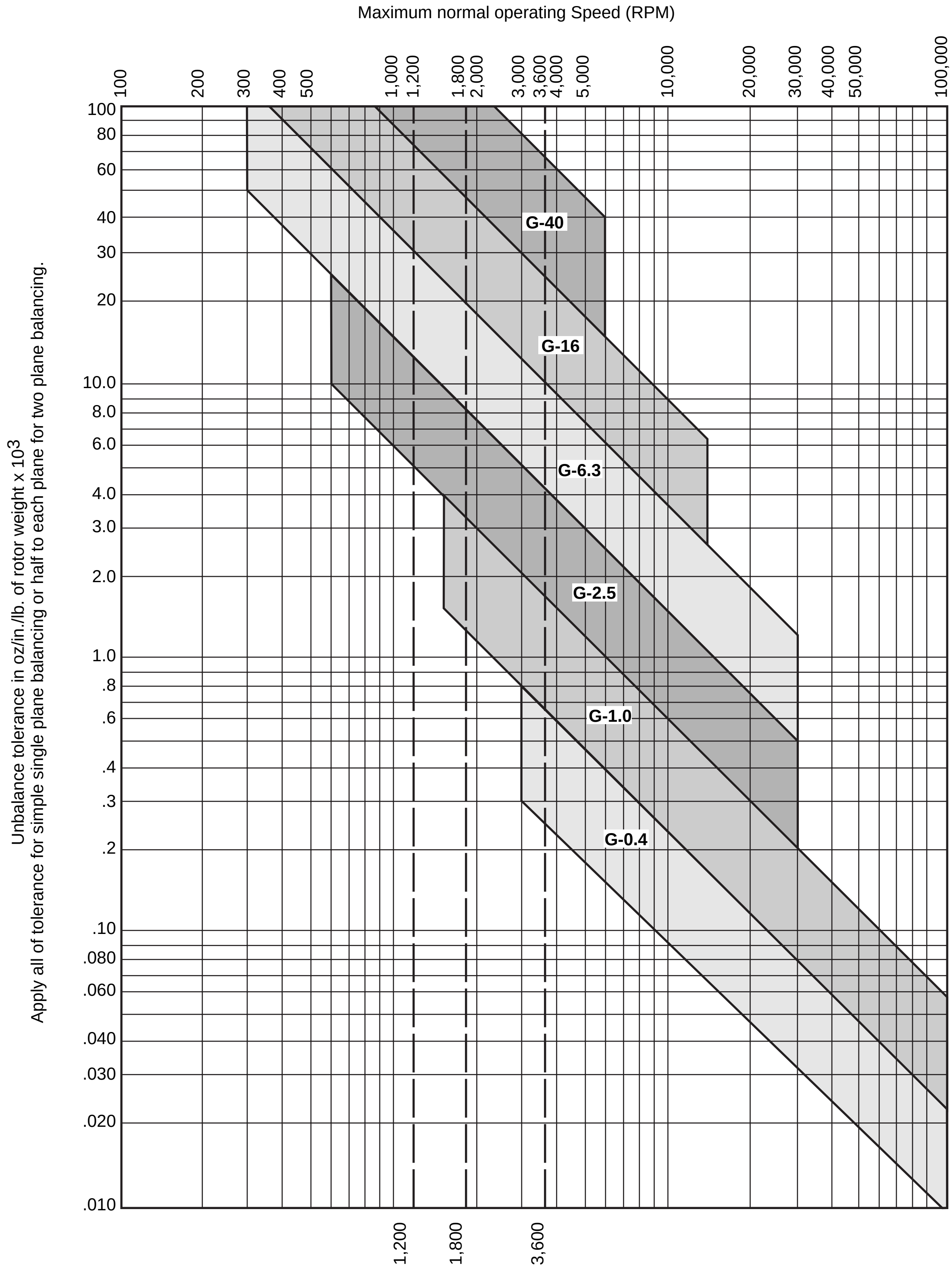
There have been a number of balancing tolerances developed over the years; a few of these are: ISO Standards (International Standards Organization), VDI Standards (Society of German Engineers), ANSI Standards (American National Standards Institute), and the Military Standards (MIL-STD-167). All of these standards share a common objective of developing a smoother running machine. In the final analysis, however, all balancing standards specify an allowable eccentricity, or offset weight distribution, from the rotating centerline.

The most common referenced standard other than API is the ISO Standard. This Standard provides a series of rotor classifications as a direct plot of residual unbalance per unit of rotor mass versus service speed (see Figure 3-2). For this specification, turbomachinery rotors are assigned the Grade of 2.5. This ISO Grade of 2.5 equates to an API upper limit allowable unbalance of about 15*W*/*N* and a lower limit of about 6*W*/*N*. This range has been found to be unsatisfactory for most turbomachinery applications. To achieve the same allowable residual unbalance level as the 4*W*/*N* API Standard would require an ISO Grade of 0.7. This comparison can be graphically illustrated as shown in the shaft centerline unbalance orbits of Figure 3-3.

Although the API 4*W*/*N* balance tolerance is significantly tighter than that of ISO Grade G-2.5, this tighter tolerance is just as easy for an experienced balancing machine operator to achieve and requires little additional time.

3.2.5 CAUSES OF UNBALANCE

There are many reasons for unbalance in a rotor. The most common causes of rotor unbalance are the following:



(Based on VDI Standards by the Society of German Engineers, Oct. 1963)

Figure 3-2—ISO Unbalance Tolerance Guide for Rigid Rotors

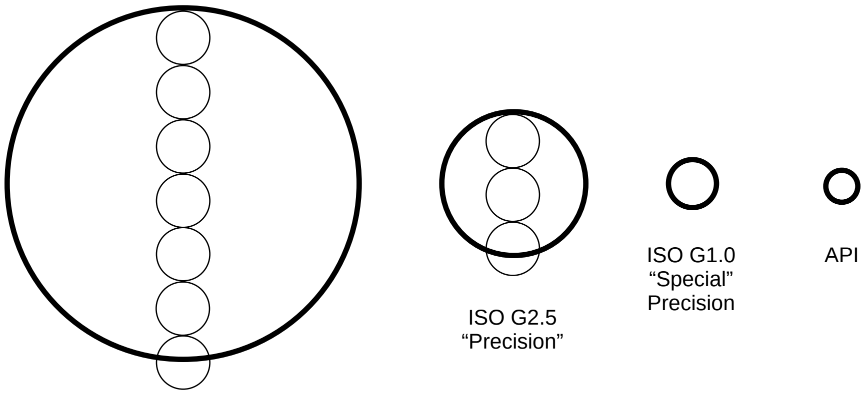


Figure 3-3—Shaft Centerline Unbalance Orbits
(based on ISO and API standards)

a. Nonhomogeneous material: On occasion, cast rotors, such as pump impellers, will have blow holes or sand traps which result from the casting process. This condition can also be caused by porosity in the rolled or forged material for shafting and impeller disks. These areas are undetectable through visual inspection. Nonetheless, the void created may cause a significant unbalance.
b. Eccentricity: This exists when the geometric centerline of a rotating assembly does not coincide with its rotating centerline. The rotor itself may be perfectly round; however, for one reason or another, the center of rotation has been displaced.
c. Lack of rotor symmetry: This is a condition that can result from a casting core shift as in a pump impeller.
d. Distortion: Although a part may be reasonably well balanced following manufacture, there are many influences which may serve to alter its original balance. Common causes of such distortion include stress relief and thermal distortion. Impeller stress relieving can be a problem with rotors which have been weld fabricated. Any part that has been shaped by pressing, drawing, bending, extruding, and so on, without stress relief, will naturally have high internal stresses. If the rotor or component parts are not stress relieved during manufacture, they may undergo stress relief naturally over a period of time, and as a result, the rotor may distort slightly to take a new shape.

Distortion that occurs with a change in temperature is called *thermal distortion*. Although all metals expand when heated, most rotors, due to minor imperfections and uneven heating, will expand unevenly causing distortion. This distortion is quite common on machines that operate at elevated temperatures (that is, induced draft boiler fans and steam turbines). For this reason steam turbine shafting is often subjected to a heat stability test.
e. Stacking errors: Slight variations in mounting or *cocking* a shrink-fit disk or impeller on its shaft may result in unbalance. This can also result from the stack-up of machining tolerances in a rotating assembly or from unaccounted for keys or other missing components.
f. Bent rotor shaft: This condition shifts the whole rotor off-center from the axis of rotation.
g. Corrosion and erosion: Many rotors, particularly those involved in material handling processes, are subjected to corrosion, erosion, abrasion, and wear. If the corrosion or wear does not occur uniformly, unbalance will result.

In summary, all of the preceding causes of unbalance can exist to some degree in a rotor. However, the vector summation of all unbalance can be considered as a concentration at a point called the *heavy spot*. Balancing is thus the technique for determining the amount and location of this heavy spot so that an equal amount of weight can be removed at this location, or an equal amount of weight added 180 degrees opposite of the heavy spot.

3.3 Balancing Machines

3.3.1 GENERAL

In choosing a balancing machine for a given rotor, care must be taken to ensure that the rotor weight is matched to the balancing machine capability and will be of sufficient sensitivity to provide good residual unbalance data (in other words, is the balancing machine capable of providing the unbalance tolerance required?).

The balancing machine drive system must also be of sufficient horsepower to bring the rotor up to the desired balance speed. This is of particular importance with large steam turbine and centrifugal fan rotors.

There are two basic types of balancing machines, namely *soft bearing* and *hard bearing* machines. The term *hard* or *soft* refers to the support system used in these machines, not to the type of bearings employed.

3.3.2 SOFT-BEARING BALANCING MACHINES

The *soft-bearing* balancing machine design employs a flexible spring support system on which the workpiece is mounted. The natural frequency of a soft-bearing support system (including the rotating assembly to be balanced) is very low, so actual balancing is done above this system's natural frequency. The unbalance in the rotor results in an unrestrained vibratory motion in the support system. This motion is normally measured with velocity transducers mechanically connected to the support system.

With soft-bearing balancing machines, different types of rotors of the same weight will produce different displacements of the vibration pickups, depending upon the configuration of the rotors to be balanced. The signals coming from the vibration pickups are dependent not only on the unbalances and on their positions, but also on the masses and moments of inertia of the rotor and its supporting system. The methods employed in a soft-bearing balancing machine are similar to those found in field balancing.

The absolute value of unbalance can be obtained only after calibrating the measuring devices to the rotor being balanced using *test masses* which constitute a known amount of unbalance. Therefore, this type of balancing machine is generally used for production applications in which many identical components are successively balanced.

3.3.3 HARD-BEARING BALANCING MACHINE

Hard-bearing balancing machines are essentially the same as soft-bearing machines except that the supports are much stiffer. This stiffness results in the critical speed, or natural frequency, of the balancing machine rotor bearing system's being well above balancing speeds. A hard-bearing balancing machine will also accept a wider range of rotor weights and configurations without requiring recalibration. This type of balancing machine measures rotor unbalance using strain-gauge transducers and, since the force that a given unbalance develops at a speed is always the same regardless of the size of the rotor, the sensing element's readouts are proportional to the rotor unbalance. Since the readout of hard-bearing machines is unbalance forces and not vibration of a spring force system, the readout will be close to the amount of actual unbalance in a properly calibrated machine. This output is not influenced by the bearing mass, rotor weight, rotor configuration, rotor moment of inertia, or windage oscillations from the rotating workpiece.

3.3.4 BALANCING MACHINE DRIVES

Balancing machines typically employ one of three different drive configurations to spin the rotating assembly. These drive mechanisms include the direct end drive, the wrap-around belt drive, and the tangential belt drive.

The direct end drive or universal joint connection is typically used with rotors having large moments of inertia or high windage losses. This drive design will transmit high torque forces for fast acceleration and safe braking. To attach the drive shaft, the rotor ends must be prepared to accept the U-joint directly or with an adaptor. With this design, the drive system becomes a part of the rotor and must be considered in the balancing accuracy of the system. Before this type of drive can be used to accurately balance a workpiece, the U-joint assembly, itself, must be balanced. The desired end result is to be able to rotate the U-joint assembly 180 degrees in relation to the workpiece without any variance in the balancing machine readout.

For rotors weighing under 2250 kilograms (5000 pounds) and with at least one smooth surface, a wrap-around belt drive works very well. For this drive configuration, 2250 kilograms (5000 pounds) is about the maximum rotor weight that will allow adequate torque transmission to bring the rotor up to the required balance speed. A 180 degree-wrap is required to provide adequate torque to rotate the rotor and keep the applied torque in the vertical plane of the balancing machine pedestal. A belt drive of this type is considerably more accurate than a direct coupled end drive in that the drive mechanism does not influence the workpiece balance.

A tangential belt drive, either under the rotor or in an over-arm configuration, is frequently used in smaller capacity, high-volume-production oriented balancing machines for rotors weighing under 450 kilograms (1,000 pounds.) This type of drive configuration is used to bring the rotor up to the required balancing speed and then the drive arm is moved away from the rotor and the unbalance data is collected.

3.3.5 BALANCING MACHINE PITFALLS

A few other points concerning balancing machines, regardless of drive type, are worth noting. One is to avoid using a machine with antifriction support bearings having diameters that are equal to the journals of the workpiece. In this instance, any imperfection that results in non-concentricity of the outer race will be interpreted as unbalance by the balancing machine electronics.

In addition, since unbalance force varies as the square of the rotor speed, the highest possible safe balance speed for a given rotor should be chosen and maintained from the start to the finish of the job. The balance speed chosen is of particular importance on steam and gas turbine rotors where the stage buckets must be seated (due to centrifugal force) in their blade-root attachments in order to achieve repeatable balance data.

Balancing machines with belt drives have also been known to introduce residual magnetism into the workpiece from friction and slippage between the belt drive system and the workpiece. The residual magnetism levels of the workpiece should always be checked after using this type of balancing machine, and the workpiece degaussed if necessary.

3.3.6 HIGH SPEED BALANCING

Generally, compressor and turbine rotors do not require high-speed (or *at speed*) balancing. This balancing method can be quite time consuming and very expensive. There are, however, conditions where high-speed balancing should be considered as follows:

a. Rotors which have exhibited high vibration as they pass through their critical speeds.
b. Rotors which accelerate slowly through their critical speeds (that is, gas turbines).
c. Rotors which are running on or near a critical speed.
d. Rotors which are very sensitive to unbalance.
e. Rotors for equipment in extremely critical services.
f. Rotors going to inaccessible locations, such as offshore.
g. Very long, flexible rotors.
h. Places where a critical rotor cannot be run in its intended casing prior to installation.

High-speed balancing is not a substitute for good slow-speed balancing procedures. Thus, the rotor should be properly slow-speed balanced before attempting a high-speed balance run. Bypassing of this procedure could result in serious damage to the rotor and/or high-speed balancing machine.

A rotor dynamics analysis of the rotor and support system should be performed prior to attempting a high-speed balance. This analysis will provide information about the predicted rotor mode shape as it passes through its critical speed(s) and about the best location for balance weights to minimize rotor vibration. Note that since the stiffness of the balancing machine bearing pedestal may vary significantly from actual field installation, the critical speed as observed in the balancing machine may differ significantly from that observed when the rotor is run in the field.

The rotor and balancing machine pedestal supports are placed in a vacuum chamber to reduce the power required to turn the rotor at higher speeds and to reduce heating from windage. Specially-manufactured oil film design bearings or *job* bearings are generally necessary to perform the balancing since the high speeds require journal bearings rather than the antifriction type used in low-speed balancing machines.

Proper conditioning of the rotor workpiece to remove all bows and distortion prior to high-speed balancing is essential. This conditioning is accomplished by spinning the rotor up and down in speed until the unbalance readout and phase angle becomes stabilized. This process may also require the application of heat to the rotor during the spinning process. The time required for this stabilization will vary widely from rotor to rotor.

3.4 Balancing Procedures

3.4.1 COMPONENT BALANCING

On flexible shaft rotors (those that operate above the first critical speed), it is vital to balance all of the major components individually before assembly. This is done because if the rotor is fully assembled, there is no way to know exactly what contribution each component part is making to the total measured unbalance vector. In addition, if a large unbalance exists in one of the major components within the rotor, the rotor shaft may flex at this point during high-speed operation and cause significant damage to the rotating and stationary parts.

Each major rotor component must be individually balanced on a precision ground mandrel (Note that expanding mandrels are not acceptable for this purpose). The balance mandrel should be ground between centers to assure concentricity of all diameters throughout its length as well as to assure a good smooth surface finish. After grinding, the mandrel must be precision balanced. A trial bias weight may be used to raise the observed residual unbalance readout of the balancing machine. The desired balance result is such that no matter at what angular location the bias weight is added, the unbalance readout is always the same. In this case the residual unbalance of the precision mandrel is as close to zero as possible.

The rotor component should always be mounted to the mandrel with an interference fit, never a sliding or loose fit. If the rotor component has a keyed fit to its shaft, then the balancing mandrel should also have a matching keyway.

After each component is shrunk on its mandrel, the axial and radial runouts should be checked to ensure that the mounted impeller or hub is not cocked on its mandrel prior to component balancing. As a general rule, runouts should not exceed 0.16 mm/meter (0.002 inch per foot) of diameter.

3.4.2 PROGRESSIVE COMPONENT STACK BALANCING

After individual balancing of all major rotor components, the rotor must be progressively *stack* balanced as each major component is assembled onto the rotor shaft.

Progressive or stack balancing is necessary due to the deformation of components during assembly. Components with unequal stiffness in all planes, such as those with single keyways (as shown in Figure 3-4), may deform when shrunk onto the rotor shaft. For such components, considerable deformation and resultant unbalance can occur between mandrel balancing using a light shrink fit and stack balancing on the job shaft with a heavy shrink fit.

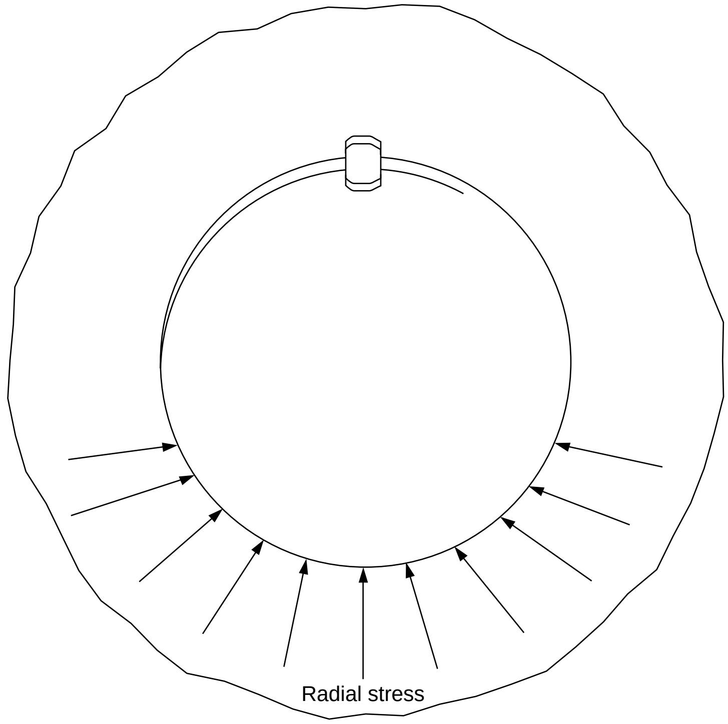


Figure 3-4—Effect of Single Key on Wheel Stiffness

Progressive balancing is accomplished by stacking no more than two rotor components at a time onto the rotor shaft. Component axial and radial runouts should be checked against mandrel runouts as each component is *stacked*. In general, the stacked component runouts should match those runouts recorded with the components on the mandrel.

As each rotor component is stacked into position and the runouts checked as acceptable, the rotating assembly is to be placed in the balancing machine and *trim* balanced (if required) as necessary to achieve the balance tolerance. Balance weight correction is to be performed only on the most recently stacked component.

After the rotor is completely stacked, trim balancing, if required at all, should be very small to meet the tolerance of 4*W*/*N* per plane. As a general rule of thumb, the remaining residual unbalance in the rotor should not exceed two times the residual unbalance tolerance prior to trim balancing.

3.4.3 KEYS AND KEYWAYS

Keys and keyway clearances are areas that are often overlooked and yet critical to a precision balance job. All keys should have a top clearance of 0.05 mm–0.15 mm (0.004 inch–0.006 inch.) Excessive top key clearance will allow the key to move radially outward during operation, resulting in a change in unbalance. Conversely, insufficient (or zero) top key clearance may prevent the wheel from properly seating on the shaft. A *tall* key will distort the wheel bore, resulting in a change in unbalance.

The *job* keys should always be used during component balancing on mandrels. Care should also be taken to ensure that each job key is removed from its component balance mandrel and then reinstalled along with its wheel on the rotor shaft in the same location. This precaution is taken because no two keys are truly identical, and the key/wheel should be considered as a *matched set* after component balancing on a mandrel.

During progressive component stack balancing, all empty shaft keyways must be filled with fully crowned half-keys to ensure that unbalance due to unfilled keyways is not compensated for in trim balancing stacked components. The crown of the half-key must match the curvature of the rotor shaft circumference.

3.4.4 RESIDUAL UNBALANCE TEST

After completion of the final balancing of the rotating assembly, and before removing the rotor from the balancing machine, a residual unbalance test should be performed to verify that the residual unbalance of the rotor is within the

4*W*/*N* tolerance. This test is performed to ensure that the balancing machine readout is correct because balancing machine calibration shift and operator error can occur.

The API Residual Unbalance Test, as shown in Appendix 3B, leaves no doubt as to the amount and location of the assembled rotors residual unbalance. In addition, since the test is performed with known values for unbalance, it is not necessarily so important that the balancing machine provide true calibrated accuracy, but only that the machine be consistent.

This test is accomplished by marking the rotor in equally spaced increments in each correction plane. A trial weight and radius is then selected that will provide approximately two times the 4*W*/*N* residual unbalance tolerance (four times 4*W*/*N* for soft bearing balancing machines). The trial weight should first be positioned at the heavy spot on the rotor (if known) to assist in selecting the proper readout scale on the balancing machine. The *heavy spot* location on the rotor is then considered as the *zero* point on the rotor for the polar plot.

The rotor is then run up to test speed, and the balancing machine amplitude and trial weight location is measured and recorded on polar graph paper (Note that the data to be recorded on the polar plot is balancing machine amplitude versus the angular location of the trial weight, not the balancing machine phase angle). This test is repeated for all trial weight positions in each balance plane of the rotor. Each plane's polar plot of balancing machine amplitude versus the trial weight location should approximate a true circle that encircles the center of the polar plot. If the plot does not approximate a true circle and/or encircle the center of the polar plot, then either the residual unbalance is in error due to inadequate sensitivity of the balancing machine, or the trial weight is smaller than the residual unbalance indicating that the rotor is not balanced correctly.

Careful placement of the known unbalance at the correct radius at each interval is essential for this test.

The polar point plotted for run-test number one should repeat at the end of the test indicating that the balancing machine is reading out consistently. A balancing machine that will not read out consistently for two identical runs cannot be used to determine true residual unbalance.

3.4.5 CHECK BALANCING

Once a rotor has been balanced in accordance with the above outlined procedures, further balancing should not be required. Far too often rotors that have been properly balanced to the correct tolerances are removed from storage for *check balancing* prior to their installation. During this check balancing procedure it is possible that the rotor will be found to be out of tolerance and, therefore, will be *rebalanced*.

Unfortunately, what the balancing machine operator does not realize is that he or she has just balanced out a temporary rotor bow resulting from long-term horizontal storage of the rotor. This temporary bow, after installation into the machine, may straighten itself after being placed into operation and thus become *unbalanced*, resulting in excessive vibration.

A rotor that has been properly stack-component balanced and documented as such, should *never* be rebalanced prior to its installation unless obvious damage or other sound justification is apparent. If the *check balance* of a rotor prior to its installation reveals an out-of-tolerance residual unbalance condition, then a thorough inspection of the rotor should be performed and additional data (that is, rotor runout maps) should be measured and recorded to ascertain the problem with the rotor. If the problem cannot be located and resolved, then the rotor should be totally disassembled and the re-stack component balanced as previously specified.

3.4.6 ACHIEVABLE RESIDUAL UNBALANCE

On many high-speed, light-weight rotors, the question often arises whether commercial balancing machines can achieve the API requirement of 4*W*/*N* for residual unbalance. By balancing the rotor components down until an unsteady phase angle is achieved (indicating the sensitivity limit of the balancing machine) and then utilizing a *bias weight*, the theoretical balancing machine tolerance of 25 microinches (or 0.05 mils peak-to-peak) can be exceeded. A bias weight simply raises the unbalance level to within balancing machine sensitivity. By moving this weight from the *heavy spot* to the *light spot* and noting the difference in the readouts, the residual unbalance can be brought significantly lower than 25 microinches.

In utilizing a bias weight, extreme care must be exercised in keeping the weight constant and in placing the weight at the correct radial and angular location. On high-speed/light-weight rotors, slight variations in the amount or location of the bias weight will have a significant impact on the residual unbalance map.

3.4.7 FIELD BALANCING

In some cases, field balancing has been found to be necessary. This is typical of very large steam turbines (over 50 MW) and for field-erected equipment that does not lend itself to shop balancing. This, however, should be considered as a last resort for high-speed rotors.

Field balancing attempts to correct for the combined effect of misalignment, seal rubs, foundation resonance, and rotor unbalance. This approach does not address the true cause of the excitation forces on the rotor. Generally, only end-plane balance corrections are available, which further limits the effectiveness of this balancing method.

While field balancing has been done successfully, it has a fairly high failure rate in rotors with multiplane correction points and may not correct the problem despite the best of efforts.

APPENDIX 3A—API *STANDARD PARAGRAPHS* SECTION 2.8.5 ON VIBRATION AND BALANCE

The following are unannotated excerpts from API *Standard Paragraph* 2.8.5:

2.8.5 VIBRATION AND BALANCE

2.8.5.1 Major parts of the rotating element, such as the shaft, balancing drum, and impellers, shall be dynamically balanced. When a bare shaft with a single keyway is dynamically balanced, the keyway shall be filled with a fully crowned half-key. The initial balance correction to the bare shaft shall be recorded. A shaft with keyways 180 degrees apart but not in the same transverse plane shall also be filled as described above.

2.8.5.2 The rotating element shall be multiplane dynamically balanced during assembly. This shall be accomplished after the addition of no more than two major components. Balancing correction shall only be applied to the elements added. Minor correction of other components may be required during the final trim balancing of the completely assembled element. On rotors with single keyways, the keyway shall be filled with a fully crowned half-key. The weight of all half-keys used during final balancing of the assembled element shall be recorded on the residual unbalance work sheet (see Appendix B). The maximum allowable residual unbalance per plane (journal) shall be calculated as follows:

$$U_{\max} = 4W/N$$

In SI units, this translates to

$$U_{\max} = 6350W/N$$

Where:

U_{max} = residual unbalance, in ounce-inches (gram- millimeters).
W = journal static weight load, in pounds (kilograms).
N = maximum continuous speed, in revolutions perminute.

When spare rotors are supplied, they shall be dynamically balanced to the same tolerances as the main rotor. [2.8.3.4]

2.8.5.3 After the final balancing of each assembled rotating element has been completed, a residual unbalance check shall be performed and recorded in accordance with the residual unbalance worksheet (see Appendix B).

2.8.5.4 High-speed balancing (balancing in a high-speed balancing machine at the operating speed) shall be done only with the purchaser's specific approval. The acceptance criteria for this balancing shall be mutually agreed upon by the purchaser and the vendor.

2.8.5.5 During the shop test of the machine, assembled with the balanced rotor, operating at its maximum continuous speed or at any other speed within the specified operating speed range, the peak-to-peak amplitude of unfiltered vibration in any plane, measured on the shaft adjacent and relative to each radial bearing, shall not exceed the following value or 2.0 mils (50 micrometers), whichever is less:

$$A = (12{,}000/N)^{0.5}$$

In SI units,

$$A = 25.4(12{,}000/N)^{0.5}$$

Where:

A = amplitude of unfiltered vibration, in micrometers (mil) true peak-to-peak.
N = maximum continuous speed, in revolutions per minute.

At any speed greater than the maximum continuous speed, up to and including the trip speed of the driver, the vibration shall not exceed 150 percent of the maximum value recorded at the maximum continuous speed. [2.8.5.8, 2.9.3.1, 4.3.3.1.11, 4.3.3.3.1, 4.3.3.3.2]

Note: These limits are not to be confused with the limits specified in 2.8.3 for shop verification of unbalance response.

2.8.5.6 Electrical and mechanical runout shall be determined and recorded by rolling the rotor in V blocks at the journal centerline while measuring runout with a noncontacting vibration probe and a dial indicator at the centerline of the probe location and one probe-tip diameter to either side.

2.8.5.7 Accurate records of electrical and mechanical runout, for the full 360 degrees at each probe location, shall be included in the mechanical test report.

2.8.5.8 If the vendor can demonstrate that electrical or mechanical runout is present, a maximum of 25 percent of the test level calculated from Equation 6 or 6.5 micrometers (0.25 mil), whichever is greater, may be vectorially subtracted from the vibration signal measured during the factory test.

APPENDIX 3B—API *STANDARD PARAGRAPHS* EXCERPTS FROM APPENDIX B—PROCEDURE FOR DETERMINATION OF RESIDUAL UNBALANCE (R20)

The following are unannotated excerpts from API *Standard Paragraphs*, Appendix B (R20):

B.1 Scope

This appendix describes the procedure to be used to determine residual unbalance in machine rotors. Although some balancing machines may be set up to read out the exact amount of unbalance, the calibration can be in error. The only sure method of determining residual unbalance is to test the rotor with a known amount of unbalance.

B.2 Definition

Residual unbalance is the amount of unbalance remaining in a rotor after balancing. Unless otherwise specified, it shall be expressed in ounce-inches or gram-millimeters.

B.3 Maximum Allowable Residual Unbalance

B.3.1 The maximum allowable residual unbalance per plane shall be calculated using Equation 5 in 2.8.5.2 of this standard.

B.3.2 If the actual static weight load on each journal is not known, assume that the total rotor weight is equally supported by the bearings. For example, a two-bearing rotor weighing 2720 kilograms (6000 pounds) would be assumed to impose a static weight load of 1360 kilograms (3000 pounds) on each journal.

B.4 Residual Unbalance Check

B.4.1 GENERAL

B.4.1.1 When the balancing-machine readings indicate that the rotor has been balanced to within the specified tolerance, a residual unbalance check shall be performed before the rotor is removed from the balancing machine.

• **B.4.1.2** To check residual unbalance, a known trial weight is attached to the rotor sequentially in six (or twelve, if specified by the purchaser) equally spaced radial positions, each at the same radius. The check is run in each correction plane, and the readings in each plane are plotted on a graph using the procedure specified in B.4.2.

B.4.2 PROCEDURE

B.4.2.1 Select a trial weight and radius that will be equivalent to between one and two times and the maximum allowable residual unbalance [that is, if U_{max} is 1440 gram-millimeters (ounce-inches), the trial weight should cause 1440–2880 gram millimeters (2–4 ounce-inches) of unbalance.]

B.4.2.2 Starting at the last known heavy spot in each correction plane, mark off the specified number of radial positions (six or twelve) in equal (60- or 30-degree) increments around the rotor. Add the trial weight to the last known heavy spot in one plane. If the rotor has been balanced very precisely and the final heavy spot cannot be determined, add the trial weight to any one of the marked radial positions.

B.4.2.3 To verify that an appropriate trail weight has been selected, operate the balancing machine and note the units of unbalance indicated on the meter. If the meter pegs, a smaller trial weight should be used. If little or no meter reading results, a larger trial weight should be used. Little or no meter reading generally indicates that the rotor was not balanced precisely enough or that the balancing machine is not sensitive enough. If this occurs, the balancing machine can be checked for sensitivity by using the procedure outlined in B.5 and Figure B-1. A completed example is shown in Figure B-2.

B.4.2.4 Locate the weight at each of the equally spaced positions in turn, and record the amount of unbalance indicated on the meter for each position. Repeat the initial position as a check. All verification shall be performed using only one sensitivity range on the balance machine.

B.4.2.5 Plot the readings on the residual unbalance work sheet and calculate the amount of residual unbalance (see Figure B-3). The maximum meter reading occurs when the trial weight is added at the rotor's heavy spot; the minimum reading occurs when the trial weight is opposite the heavy spot. Thus, the plotted readings should form an approximate circle (see Figure B-4). An average of the maximum and minimum meter readings represents the effect of the trial weight. The distance of the circle's center from the origin of the polar plot represents the residual unbalance in that plane.

B.4.2.6 Repeat the steps described in B.4.2.1 through B.4.2.5 for each balance plane. If the specified maximum allowable residual unbalance has been exceeded in any balance plane, the rotor shall be balanced more precisely and checked again. If a correction is made in any balance plane, the residual unbalance check shall be repeated in all planes.

Equipment (Rotor) No.: __________

Purchase Order No.: __________

Correction Plane (inlet, drive-end, etc.—use sketch): __________

Balancing Speed: __________ rpm

N—Maximum Allowable Rotor Speed: __________ rpm

W—Weight of Journal (Closest to this correction plane): __________ lbs

Umax = Maximum Allowable Residual Unbalance =
4 x WIN (6350 WIN)
4 x _________ lbs/____________rpm __________ oz.-in. (gm-mm)

Trial unbalance (2 x Umax) __________ oz.-in. (gm-mm)

R—Radius (at which weight will be placed): __________ inches

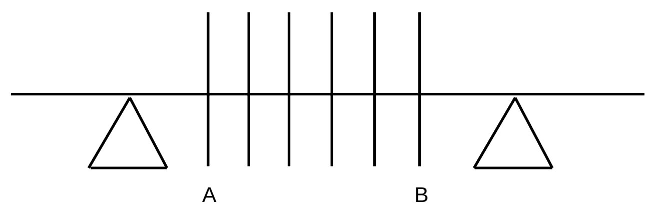
Trial Unbalance Weight = Trial Unbalance/R
_________oz.-in./_________inches = __________ oz. (gm)

Conversion Information: 1 ounce = 28.375 grams

Test Data

Position	Trial Weight Angular Location	Balancing Machine Amplitude Readout
1		
2		
3		
4		
5		
6		
7		

Rotor Sketch



C-101

Test Data—Graphic Analysis

Step 1: Plot data on the polar chart (Figure C-4 continued). Scale the chart so the largest and smallest amplitude will fit conveniently.

Step 2: With the compass, draw the best fit circle through the six points and mark the center of this circle.

Step 3: Measure the diameter of the circle in units of scale chosen in Step 1 and record. __________ units

Step 4: Record the trial unbalance from above. __________ 0z.-in. (gm-mm)

Step 5: Double the trial unbalance in Step 4 (may use twice the actual residual unbalance). __________ oz.-in. (gm-mm)

Step 6: Divide the answer in Step 5 by the answer in Step 3. __________ Scale Factor

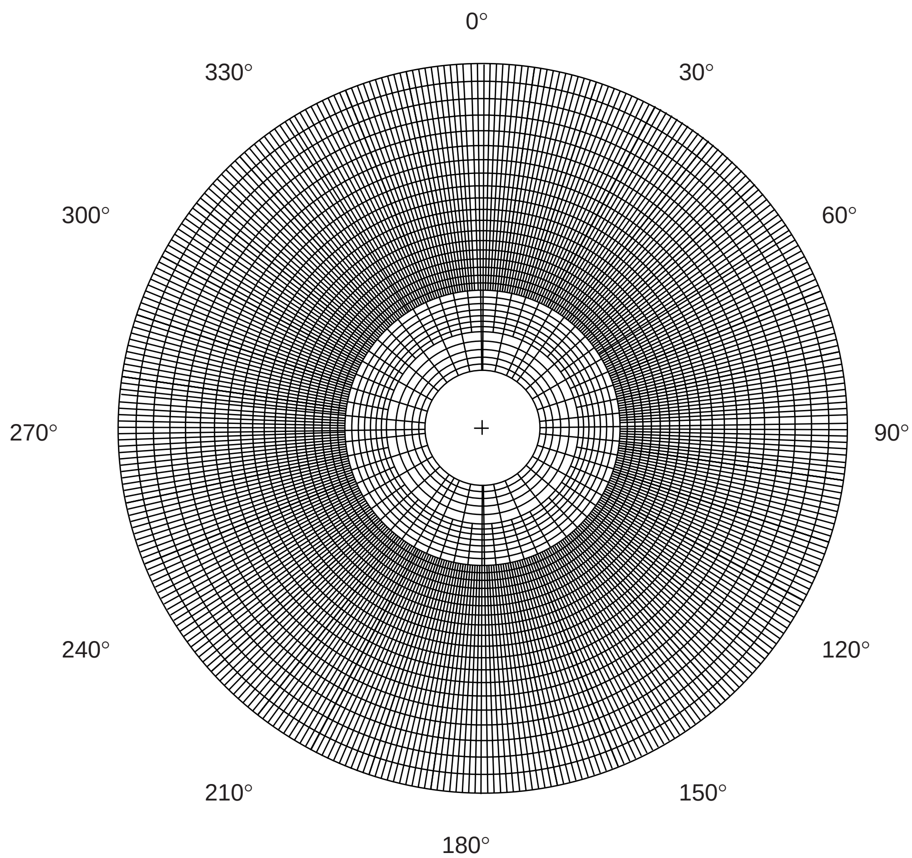
You now have a correlation between the units on the polar chart and the gm-in. of actual balance.

Notes:

1 The trial weight angular location should be referenced to a keyway or some other permanent marking on the rotor.

2 The balancing machine amplitude readout for Position 7 should be the same as Position 1 indicating repeatability. Slight variations may result from imprecise positioning of the trial weight.

Figure B-3—Residual Unbalance Work Sheet



The circle you have drawn must contain the origin of the polar chart. If it doesn't, the residual unbalance of the rotor exceeds the applied test unbalance.

If the circle does contain the origin of the polar chart, the distance between origin of the chart and the center of your circle is the actual residual unbalance present on the rotor correction plane. Measure the distance in units of scale you choose in Step 1 and multiply this number by the scale factor determined in Step 6. Distance in units of scale between origin and center of the circle times scale factor equals actual residual unbalance.

Record actual residual unbalance ____(oz.-in.)(gm-mm)

Record allowable residual unbalance (from Figure B-3)___________________________(oz.-in,)(gm-mm)

Correction plane _______________________ for Rotor No. ___________________(has/has not) passed.

By____Date___________________________________

Figure B-3—Residual Unbalance Worksheet (Continued)

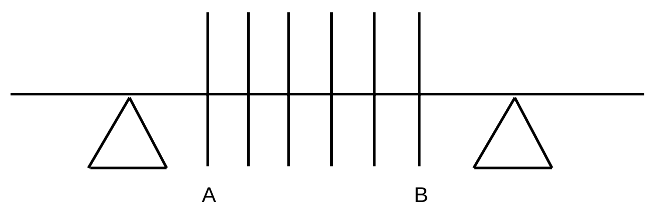
Equipment (Rotor) No.:	C-101	
Purchase Order No.:		
Correction Plane (inlet, drive-end, etc.—use sketch):	A	
Balancing Speed:	800	rpm
N—Maximum Allowable Rotor Speed:	10.000	rpm
W—Weight of Journal (Closest to this correction plane):	908	lbs
Umax = Maximum Allowable Residual Unbalance = 4 x WIN (6350 WIN) 4 x 908 lbs/ 10.000 rpm	036	oz.-in. (gm-mm)
Trial unbalance (2 x Umax)	0.72	oz.-in. (gm-mm)
R—Radius (at which weight will be placed):	6.875	inches
Trial Unbalance Weight = Trial Unbalance/R 0.72 oz.-in./ 6.875 inches =	0.10	oz. (gm)

Conversion Information: 1 ounce = 28.375 grams

Test Data

Position	Trial Weight Angular Location	Balancing Machine Amplitude Readout
1	0°	16.2
2	60°	12.0
3	120°	12.5
4	180°	17.8
5	240°	24.0
6	300°	23.0
7	0°	16.2

Rotor Sketch



C-101

Test Data—Graphic Analysis

Step 1: Plot data on the polar chart (Figure C-4 continued). Scale the chart so the largest and smallest amplitude will fit conveniently.

Step 2: With the compass, draw the best fit circle through the six points and mark the center of this circle.

Step 3: Measure the diameter of the circle in units of scale chosen in Step 1 and record.	35	units
Step 4: Record the trial unbalance from above.	0.72	0z.-in. (gm-mm)
Step 5: Double the trial unbalance in Step 4 (may use twice the actual residual unbalance).	1.44	oz.-in. (gm-mm)
Step 6: Divide the answer in Step 5 by the answer in Step 3.	0.041	Scale Factor

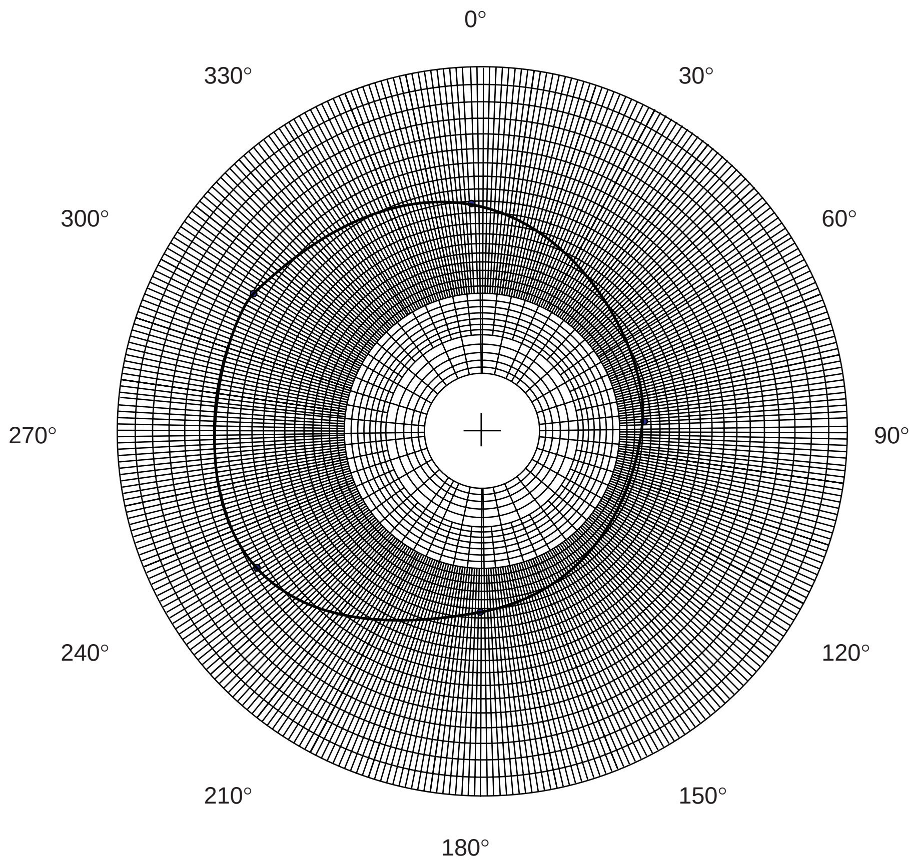
You now have a correlation between the units on the polar chart and the gm-in. of actual balance.

Notes:

1 The trial weight angular location should be referenced to a keyway or some other permanent marking on the rotor.

2 The balancing machine amplitude readout for Position 7 should be the same as Position 1 indicating repeatability. Slight variations may result from imprecise positioning of the trial weight.

Figure B-3—Residual Unbalance Work Sheet (Continued)



The circle you have drawn must contain the origin of the polar chart. If it doesn't, the residual unbalance of the rotor exceeds the applied test unbalance.

If the circle does contain the origin of the polar chart, the distance between origin of the chart and the center of your circle is the actual residual unbalance present on the rotor correction plane. Measure the distance in units of scale you choose in Step 1 and multiply this number by the scale factor determined in Step 6. Distance in units of scale between origin and center of the circle times scale factor equals actual residual unbalance.

Record actual residual unbalance 6.5 (0.041) = 0.27 (oz.-in.)(gm-mm)

Record allowable residual unbalance (from Figure B-3) 0.36 (oz.-in,)(gm-mm)

Correction plane A for Rotor No. C-101 (has/has not) passed.

By ________ Date 11-16-92

Figure B-3—Residual Unbalance Worksheet (Continued)

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